Nonlinear Adjustment of Emerging Stock Market Returns: Symmetrical or Asymmetrical

Seyyed Ali Paytakhti Oskooe
Islamic Azad University (IAU), Oxford Branch, UK.
Email: ali.paytakhti@auo.org.uk

ABSTRACT: This study examines whether the nonlinear adjustment dynamic of stock returns to the equilibrium level in an emerging stock market is symmetrical or asymmetrical. The empirical results suggest that the data generating process of Iran stock returns series is nonlinear Smooth Transition Autoregressive (STAR) and dynamic adjustment of the stock returns to the long run equilibrium level is asymmetric. The adjustment mechanism of the Iran stock returns deviations from the equilibrium level are different in the bull and bear markets.

Keywords: Stock returns; Smooth transition; Bull and bear markets
JEL Classifications: G12; G14; G15; C22

1. Introduction
Analysis of stock prices or stock returns behaviour is one of the fundamental tasks in applied financial econometrics. In this sense, identification of stock returns movements play a crucial role in portfolio management and reducing the risk of investment in stock markets. In order to in-depth analysis of how to behave of stock returns attention to true data generating process is determinant. In recent years, concentration on the nonlinear behaviour in the modelling of financial time series has been steadily increasing in financial economics literature. In this sense, the nonlinear Smooth Transition Autoregressive (STAR) specification is able to model better the dynamic behaviour of stock returns. Because it allows for a smooth transition between regimes and symmetric or asymmetric adjustment of the stock returns above and below the equilibrium level (averaged returns in long run). Using STAR regime-switching models the nonlinear behaviour of stock return series has been documented only in few empirical studies (McMillan, 2001, 2004; Bradley and Jensen, 2004; Manzan, 2006; Bonga-Bonga and Makakabule, 2010).

Understanding the form in which the adjustment of stock returns to their long-run equilibrium takes place is an important issue in order to comprehend the behaviour of stock prices and, consequently, to provide an appropriate policy response by stock market authorities. At the same time, identifying the dynamic behaviour of stock prices or stock returns in the bull and bear markets (two stats of stock market) can be helpful in the risk management and portfolio construction.

The remainder of this paper is organised as follows: Section 2 presents the empirical methodology. The data and empirical results are introduced in section 3. The paper ends with concluding remarks.

2. Methodology
In order to examine for a potential non-linear behaviour in deviation stock returns dynamic from equilibrium level, we apply the regime switching nonlinear smooth-transition threshold model. In this approach of nonlinear modelling, the dynamic behaviour of stock returns depends on the regime that occurs at any given point in time. In this sense the statistic properties of the stock return time series, such as its mean, variance and/or autocorrelation, are different in different regimes. In the context of the regime switching models small and large deviation from equilibrium level are characterised by different dynamics.

The two regimes STAR (Smooth Transition Autoregressive) model for dynamic adjustment of stock returns which allows the autoregressive parameters to change slowly can be expressed as:
\[ rt = \phi'_0 W_t + (\phi'_1 W_t) F(S_t, \gamma, c) + \varepsilon_t, \quad \varepsilon_t \approx iid(0, \sigma_t) \]  

Where \( W_t = (1, r_{t-1}, \ldots, r_{t-p})' \) and \( \phi_0 = (\phi_{00}', \ldots, \phi_{0p}')' \) and \( \phi_1 = (\phi_{10}', \ldots, \phi_{1p}')' \) are \((p+1) \times 1\) parameter vectors. \( F(S_t, \gamma, c) \) is the transition function bounded by zero (it changes smoothly from 0 to 1 as \( S_t \) increases) and unity and \( S_t \) is the transition variable that determines the switch between regimes (see Van Dijk et al. 2002). The parameter \( c \) is the threshold and gives the location of the transition function (determines the regime that is active), while \( \gamma \) as the slope of the transition function (smoothness parameter) determines the speed of transition. Following Teräsvirta (1994) the transition variable \( S_t \) is assumed to be the lagged dependent variable.

The two common form of transition function are defined:

The logistic function: 
\[ F(S_t, \gamma, c) = (1 + \exp(-\gamma(S_t - c)))^{-1}, \quad \gamma > 0 \]  

where (1) and (2) yield the LSTAR model. The logistic function varies smoothly from 0 to 1 as the transition variable, \( S_t \) becomes increasingly larger than the threshold \( c \). In this study, two distinct regimes are defined bull markets (periods that stock returns are positive) and bear markets (periods that stock returns are negative). When the values of transition function close to zero the LSTAR specification indicates the bear market, while in the case of the values of transition function close to unity the LSTAR model describes the bull market.

Another common form of the transition function is the exponential form:
\[ F(S_t, \gamma, c) = (1 + \exp(-\gamma(S_t - c))^2 ) \]  

Where (1) and (3) give raise the ESTAR model. In this case the transition function smoothly approaches 1, the further \( S_t \) deviates (in either direction) from the threshold value \( c \). the exponential function implies symmetric adjustment in both directions of \((S_t - c)\). In contrast, the logistic function is characterized by an asymmetric adjustment of \( r_t \) to its past values depending on the transition variable, \( S_t \), being above or below the threshold \( c \). In other words, in the logistic function negative and positive deviations revert back to the fundamental at different speeds, whereas in the exponential function the speed of mean reversion is equal for all (negative and positive) deviations. More specifically, the LSTAR allows different behaviour depending on whether stock returns are positive or negative, while the ESTAR allows different behaviour to occur for large and small returns regardless of sign. In sum, the ESTAR specification implies the same dynamic adjustment for bull and bear markets.

The null hypothesis of linearity against STAR is defined as:

\[ H_0: \phi_0 = \phi_1 \quad \text{or} \quad \gamma = 0 \]

Teräsvirta (1994) argue that under both null hypotheses the test statistics are affected by the presence of nuisance parameters that complicate the derivation of the asymptotic distribution. In order to overcome this identification problem Taylor-series approximation of transition function \( F(S_t, \gamma, c) \) is employed. This allows us to derive an LM type statistic with a standard \( \chi^2 \) distribution.

A third-order Taylor-series expansion of the exponential transition function around \( \gamma = 0 \) leads to the auxiliary regression:

\[ rt = zt + zt r_t - d + zt r_t - d^2 + zt r_t - d^3 + u_t \]  

(4)
Nonlinear Adjustment of Emerging Stock Market Returns: Symmetrical or Asymmetrical

Where \( z_t = (a_0, rt_{-1}, rt_{-2}, ..., rt_{-p}) \)

In the next step in order to test for the presence of LSTAR behaviour following Enders(2010) we estimate the following auxiliary regression using the error term series from AR (p) model:

\[
\begin{align*}
\epsilon_t &= a_0 + a_1 rt_{-1} + \ldots + a_p rt_{-p} + a_{11} rt_{-1} rt_{-d} + \ldots + a_{1p} rt_{-p} rt_{-d} \\
&+ a_{21} rt_{-1} rt_{-d} + \ldots + a_{2p} rt_{-p} rt_{-d} + a_{31} rt_{-1} rt_{-d} + \ldots + a_{3p} rt_{-p} rt_{-d} + \epsilon_t \\
\end{align*}
\]

(5)

In this condition, the null hypothesis that \( \gamma = 0 \) against STAR dynamics is the joint restriction that all nonlinear terms are zero:

\[ H_0: a_{11} = \ldots = a_{1p} = a_{21} = \ldots = a_{2p} = a_{31} = \ldots = a_{3p} = 0 \]

In order to make robust inference in small samples we use the standard F-test with 3p degrees of freedom in the numerator. If the null hypothesis of linearity is rejected (it is accepted that model is nonlinear), in order to choose between the LSTAR and ESTAR forms of regime switching model we test the following restriction using an F-test:

\[ H_{0, E}: a_{31} = \ldots = a_{3p} = 0 \]

The rejection of this null hypothesis implies the model has the LSTAR form (the dynamic adjustment is asymmetric); otherwise it means that the dynamic adjustment is ESTAR.

3. Data and Preliminary Analysis

The series used in this study is the daily stock price index of Iran stock market (TEPIX) which is calculated as weighted market value of all share prices appearing on the TSE Price Board. The sample period includes 2632 observations during the period January 2, 1999 to December 30, 2009.

Market prices index are transformed to daily returns \( r = \ln(p_t / p_{t-1}) \) where \( p_t \) and \( p_{t-1} \) are stock prices index prices at date \( t \) and \( t - 1 \) respectively.

Table 1 provides the descriptive statistics (the unconditional distribution statistics) for Iran stock returns. The wide gap between the maximum (5.2581%) and minimum (-5.4530%) returns gives support to the high variability of price variation in the Iran stock market. A visual analysis of the market volatility can be seen in Figure 1. The mean of stock return series is constant, while the variance keeps changing over time. The large changes (of either sign) in stock returns tend to be followed by large changes, and small movements (of either sign) being followed by small movements. This is a property of stock returns distribution known as volatility clustering or volatility pooling (a type of heteroscedasticity) that Iran stock return series seems to exhibit. In view of the skewness and the kurtosis statistics of Iran stock returns distribution it can be concluded that the distribution of stock returns departs from normal distribution. Furthermore, according to the calculated Jarque-Bera statistics and corresponding p-value in table 1, the stock return series is not well approximated by the normal distribution. Generally speaking, in line with the findings of other empirical studies in emerging stock markets, the Iran stock return time series are characterized by some “stylized facts” such as fat tails, high peakness (excess kurtosis), skewness and volatility clustering.
4. Empirical Results
In order to identify the dynamic adjustment of Iran stock returns to the equilibrium expected rate, firstly the AR(p) model was estimated to determine the order p and obtain the residuals (εt). Based on looking at the partial autocorrelation function (PACF) and diagnostic test (Ljung-Box Q-statistics) the order of AR model was selected 10 (p=10). Because of insignificance of AR (3) to AR (7) coefficients, they are ruled out from estimated AR model. It should be noted that the delay parameter was determined 1 (d=1). Using the residuals (without any linear dependency) from estimated AR (10) model, the equation (5) was estimated as follow and based on obtained coefficients the null hypothesis of linearity was examined.

\[ \epsilon_t = a_0 + a_1 r_{t-1} + \ldots + a_{10} r_{t-10} + a_{11} r_{t-1}^2 + a_{12} r_{t-2}^2 + a_{18} r_{t-1} r_{t-8} \]
\[ + a_{19} r_{t-9} + a_{110} r_{t-10} r_{t-10} + a_{21} r_{t-1}^3 + a_{22} r_{t-2} r_{t-1}^2 + a_{28} r_{t-8}^2 + a_{29} r_{t-9} r_{t-1}^2 + a_{210} r_{t-10} r_{t-1}^2 \]
\[ + a_{31} r_{t-1}^4 + a_{32} r_{t-2}^3 r_{t-1} + a_{38} r_{t-8}^3 r_{t-1} + a_{39} r_{t-9}^3 r_{t-1} + a_{310} r_{t-10}^3 r_{t-1} + \epsilon_t \]
As it can be seen from the table, the F-statistic for the following null hypothesis equals 21.57.

\[ H_0: a_{11} = a_{12} = a_{18} = a_{19} = a_{21} = a_{22} = a_{28} = a_{29} = a_{31} = a_{32} = a_{38} = a_{39} = a_{310} = 0 \]

Since there are 15 numerator and 2607 denominator degrees of freedom, we can reject the null hypothesis at any conventional significant level (the 1 percent critical value is 2.039). This means that the underlying data generating process of the Iran stock return series is nonlinear (in particular STAR model). After identifying the threshold behaviour, we can select the LSTAR or ESTAR model based on testing following null hypothesis:

\[ H_0, E: a_{31} = a_{32} = a_{38} = a_{39} = a_{310} = 0 \]

Since the F-statistic is 16.53, the null hypothesis is rejected in favour of LSTAR behaviour. In other words, the dynamic adjustment of the Iran stock returns is asymmetric Smooth Transition Autoregressive (STAR) model.

### Table 2. The F-statistics for the linearity and model selection tests

<table>
<thead>
<tr>
<th>Test</th>
<th>Hypothesis</th>
<th>F-statistic</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( H_0 )</td>
<td>21.57</td>
</tr>
<tr>
<td></td>
<td>( H_0, E )</td>
<td>16.53</td>
</tr>
</tbody>
</table>

### 5. Concluding Remarks

Identifying the dynamic adjustment of stock returns to the equilibrium level helps to better understanding the behaviour of stock returns in different states (bull and bear markets). In view of the theoretical background, we employ the Smooth Transition Autoregressive (STAR) model to identify true data generating process of Iran stock returns series. The empirical results show that the deviations of the Iran stock returns from equilibrium level follow nonlinear path and the response of market adjustment mechanism towards over-valuation and under-valuation stock are asymmetric in nature. In this sense, the nature of adjustment is different in the bull and bear market.

### References


