An Innovative Approach to the Formation of a Progressive Taxation Probabilistic Model on Personal Incomes

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ABSTRACT

This article suggests a mathematical justification of the possibility of transition to progressive taxation on personal incomes, allowing socially redistribute the tax burden between different groups without losing the total amount of tax yield to the state budget from personal income tax. The methods of the transition from a flat to a progressive tax scale of income taxation developed. A generalized mathematical model of transition to progressive taxation, which allows evaluating all possible options for the proposed reform of flat rate transformation, is suggested. An algorithm of modeling progressive income taxation is proposed, which takes into account the likelihood of tax evasion. In the following parts of the article a probabilistic model of progressive income taxation and a mechanism to optimize the level of tax rates is formed. A linear progressive tax scale with and without tax evasion are developed based on actual statistics.

Keywords: Tax, Innovation, Probability, Method, Model
JEL Classifications: C10, H24

1. INTRODUCTION

Currently, issues related to the reform of income taxation, are of increasing importance, both in the Russian Federation as well as in various developed countries. Income tax in Russia is indeed one of the most important federal taxes and one of three tax along with value added tax and corporate profit tax, which provide the greatest revenues to the consolidated budget of the country.

In 2001, the Russian Federation refused to introduction of progressive income tax and flat rate of 13% was forced. The main argument justifying the introduction of a flat tax rate, was the idea that the big revenues will be withdrawn from the shadow economy.

Currently, the most common form of realization of the socio-orientated tax system in the world practice is the use of progressive taxation. It is explained by importance of social infrastructure branches development support which functioning is directed on improvement of conditions of formation and development of the human capital (Zaborovskaya, 2014; 2015; Rodionov et al., 2014; 2015; Rodionov et al., 2014). This confirms validity of the mechanism of progressive taxation use.

Foreign experience of income taxation are investigated in the works by Guner et al. (2014), Kanbur and Tuomala (2013), Kovárnik and Hamplová (2013), Izotova (2011), Lyashenko and Murav'eva (2014), Maslova and Kaz'min (2013), Ulez'ko and Orobinskaya (2013), Tyutyuryukov (2013), and others.

Suggestions and recommendations for reform and improvement of the income tax system are contained in works by Aliev et al. (2011), Beskorovajnaja (2012), Kashin (2012), Koren' (2014), Kosov (2014), Kushch and Yanakov (2012), Savina (2010), Tarasova and Goncharenko (2015), and others.

Issues, related to the transition to a progressive income taxation, are studied by scientists such as the Akhmadeev and Kosov (2015), Bryzgalin (2009), Gaponova and Solov'eva (2014), Grekov and Senina (2015), Panskov (2009), Polievktova (2014), Sheveleva and Zheryakova (2015), Ellaryan (2012), and others.
2. PROBLEM FORMULATION

According to the aforesaid we propose two methods of linear and nonlinear transformation and a mathematical model of transition to a progressive income taxation based on them.

The main provisions of the essence of these methods and models are as follows.

Linear transformation method allows transforming the original flat tax rate on the basis of progressive social significance single parameter - The value of the income tax rates \( n_i \) for low-income groups. According to this method the value of the tax rates for different groups of the population \( n_i \) are as follows:

\[
n_i = n_1 + \frac{S_0 (n_0 - n_1) (i-1)}{\sum_{j=2}^{m} (j-1) S_j} = n_1 + \frac{(n_0 - n_1) (i-1)}{\sum_{j=2}^{m} (j-1) n_j}
\]

(1)

Where \( S_0 \) - current tax base; \( n_u \) - Tax rate of flat tax rate, \( n_u = 13\% \); \( j \) - Index used to summarize the shares of \( n_j \) ratio; \( m \) - The number of taxation groups; \( n_j = \frac{S_j}{S_0} \) - The distribution coefficients of monetary incomes of population groups as a proportion of total revenue \( S_0 \). 

The idea of nonlinear transformation method is that it as well as the linear transformation method allows creating different tax rates for different groups, depending on their income. According to this method, special nonlinearity coefficient \( k \) is added to the formula for calculating the tax rate \( n_i \) which allows to increase the tax rate not proportionally to all groups, but with greater restatement of the tax burden on the group with the highest incomes.

The general formula for calculating tax rates by this method is the following:

\[
n_i = n_1 + \frac{(n_0 - n_1) \cdot S_0 \cdot [1 + k (i-1)] (i-1)}{\sum_{j=2}^{m} S_j (j-1) \cdot [1 + (j-1) \cdot k]}
\]

(2)

\[
n_i = n_1 + \frac{(n_0 - n_1) \cdot [1 + k (i-1)] (i-1)}{\sum_{j=2}^{m} \eta_j (j-1) \cdot [1 + (j-1) \cdot k]}
\]

(3)

Where \( \tau = \frac{C}{C_0} \) - The ratio of the planned increase in tax yields; \( C_0 \) - The total yields from income tax at a flat rate; \( C \) - Total yields from personal income tax under the progressive scale.

This model allows to evaluate different options for the proposed reform of transformation flat into progressive rate. There in before tax rates (Equation 3) vary depending on the source of external conditions, as well as the goals and challenges faced by the public authorities, developing tax policy in the short and medium term.

However, despite the various options for adjustment of tax coefficients (Equation 3), the proposed model does not take into account the probability of non-payment of income tax by raising the tax rates - transition from 13% of the linear rate to a higher rate of taxation for the affluent.

In accordance with the above, the purpose of this study is to develop mathematical tools to build a probabilistic model of taxation, which more accurately reflects the process of taxation and addresses the problem of tax evasion.

3. PROBLEM SOLUTION

3.1. Development of an Algorithm for Progressive Model of Income Taxation Constructing, Taking into Account the Likelihood of Tax Evasion

The problem of tax evasion is urgent and cause interest among scientists and economists. Questions to avoid paying taxes are considered in works by Feldstein (1999), McGuire et al. (2014), Kubick et al. (2015), and others.

The following problem is stated in the formation of a progressive income tax: To find such tax rates \( n_i \), that the amount of tax yields was the highest under the condition that the probability of actual payment of taxes depends on the level of tax rates.

Let us introduce the \( p_i \) coefficient characterizing what part of the income tax will be paid by taxpayers in the real value of income falling within the \( i^{th} \) interval, and therefore falls under the tax rate \( n_i \). These coefficients \( 0 \leq p_i \leq 1 \) can be determined from statistical data on taxation, calculated by the Federal State Statistics Service of the Russian Federation, the Ministry of Finance or the Ministry for Taxes and Levies of the Russian Federation.

Further \( p_i \) coefficients will be called income tax payment probabilities by taxpayers.

Let us assume that a taxpayer pays tax with probability \( p_i \), or doesn’t pay it at all. It is notable that the option of partial payment of the tax, i.e., display and payment only of a part of the income is not considered in this simple probabilistic model.
Under this assumption, the formula calculating the total yield from income tax, taking into account the probability of paying it with the introduction of a progressive scale will be:

\[ C = \sum_{i=1}^{m} S_i \cdot n_i \cdot p_i \] (4)

This formula allows to estimate the amount of tax yields, taking into account the probabilistic nature of the actual payment of the tax.

It is important to note, that the higher is tax rate, the less likely the taxpayer will pay tax at such a high rate.

Suppose \( p_i \) and \( n_i \) are related by:

\[ p_i = 1 - b_i \cdot n_i \] (5)

For example, if the tax rate is not great, \( n_i = 0.1 \), then the probability of paying it considering (5) would be big, \( p_i = 0.9 \); if the rate \( n_i = 0.6 \), then the probability is reduced to \( p_i = 0.4 \).

The presence of probability \( p_i \) in the Formula (4) prevents “voluntary” increase in tax rates \( n_i \), because this would immediately lead to lower tax probability \( p_i \).

The presence of divergent trends in values \( n_i \) and \( p_i \) gives hope to the possibility of a correct formulation of solutions to optimize the tax scale parameters.

Formula (4) with (5) takes the form:

\[ C = \sum_{i=1}^{m} S_i \cdot n_i \cdot (1 - n_i) \] (6)

Thus, the optimization problem comes down to picking \( n_i \) tax rates so that to provide maximum values of tax yields \( C_{\text{max}} \):

\[ C_{\text{max}} = \max_{n_i} \sum_{i=1}^{m} S_i \cdot n_i \cdot (1 - n_i) \] (7)

As a result, the problem reduces to finding the maximum of a function \( C \) with many variables \( n_i \). Under the rules of search for several variables extremum function we have:

\[
\begin{align*}
\frac{\partial C}{\partial n_1} &= S_1 (1 - 2n_1) = 0 \\
\frac{\partial C}{\partial n_2} &= S_2 (1 - 2n_2) = 0 \\
&\qquad\vdots \\
\frac{\partial C}{\partial n_m} &= S_m (1 - 2n_m) = 0
\end{align*}
\] (8)

Therefore, optimal formulated values \( n_i^* \), that provide maximum for the amount of tax levies \( C_{\text{max}} \), will be:

\[ n_1^* = n_2^* = \ldots = n_m^* = 0.5 \]

That is, the optimal tax scale in this simplest hypothetical case of a flat rate with the value \( n_i = 0.5 \); it gives more tax yield than any other scale.

This finding explains the transition in 2001 to a flat tax rate, as the economy and the tax system at the time led to greater probabilities of tax evasion.

Let us consider refined, more realistic, probabilistic model of a progressive tax system.

Assume that \( p_i \) and \( n_i \) are connected by the relation:

\[ p_i = 1 - b_i \cdot n_i \] (9)

Where \( b_i \) - weights; \( b_i = \frac{1 - p_i}{n_i} \) can be determined on the basis of statistical data.

Examples of graphs of (9) for some values of \( b_i \) are shown in Figure 1.

Considering (9), the Formula (4) takes shape:

\[ C = \sum_{i=1}^{m} S_i \cdot n_i \cdot (1 - b_i n_i) \] (10)

And the problem of optimization (10) reduces to finding the maximum of a function \( C_{\text{max}} \) with many variables \( n_i \):

\[ C_{\text{max}} = \max_{n_i} \sum_{i=1}^{m} S_i \cdot n_i \cdot (1 - b_i n_i) \] (11)

Under the rules of several variables extremum function search, we have:

\[
\begin{align*}
\frac{\partial C}{\partial n_1} &= S_1 (1 - 2b_1 n_1) = 0 \\
\frac{\partial C}{\partial n_2} &= S_2 (1 - 2b_2 n_2) = 0 \\
&\qquad\vdots \\
\frac{\partial C}{\partial n_m} &= S_m (1 - 2b_m n_m) = 0
\end{align*}
\] (12)

Thus, optimal values for \( n_i^* \) rates will be:

\[ n_1^* = n_2^* = \ldots = n_m^* = 0.5 \]
The developed mathematical tools, including a probabilistic approach to the problem of registration of tax evasion, as well as the proposed adjustment mechanism of progressive scale obtained by linear (nonlinear) transformation, let us talk about the creation of a probabilistic model of incomes taxation of the population.

The advantages of this model is that it allows to transform a progressive tax scale to clarified scale taking into account the additional possibilities of tax evasion associated with a change in tax rates as a result of progression in the income taxation. In accordance with the proposed model it is possible to find the optimal tax rates obtained on the basis of a probabilistic approach to the payment of taxes.

The typical form of adjusted progressive scale shown in Figure 2.

The solid line shows the original scale $I$, the dotted line - adjusted scale $I^*$ which results the optimization of tax rates $n_j$ according to Formula (13).

The above chart shows the mathematical recommendations to maximize tax levies, taking into account the probability of their payment $p_i$: To slightly lower tax rates for groups with higher incomes and slightly increase to low-income groups.

From the perspective of social orientation, this recommendation should not be done literally, i.e., for low-income groups unadjusted tax rate should be left. At the same time, for better personal income tax levies, the recommendations should be applied to groups with high incomes: It is advisable to adjust (decrease) the tax rate to compensate negative consequences of possible non-payment at higher rates. The amount of these adjustments is determined on the basis of solving optimization problems using Formula (13).

$$n_1 = \frac{1}{2b_1} ; \quad n_2 = \frac{1}{2b_2} ; \quad n_m = \frac{1}{2b_m} \quad (13)$$

It is worth noting, that the accuracy of adjusting the values of optimal tax rates depends on the reliability of statistical data for $p_i$. The assessment of the shadow component calculations on the proposed probabilistic model shows that the adjustment of the scale leads to a nonlinearity coefficient $k \approx 0.05$ and the adjustment coefficients tax on the value of $\approx 5-10\%$ for the extreme groups.

3.2. Formation of a Probabilistic Model for Progressive Income Taxation and Optimization of Tax Rates Levels

The following problem should be stated: It is needed to build a progressive scale of taxation that guarantees the same level of tax levies, as in the flat rate, but takes into account the probability of evasion of income tax. As in the previous section, a hypothesis is the rule that the higher the tax rate, the less the likelihood that the income tax will be paid, so the relationship (9).

In accordance with the Formula (4) the amount of tax levies in the probabilistic model is developed.

$$C = \sum_{i=1}^{m} S_i \cdot n_i \cdot p_i \quad (14)$$

Or considering (9):

$$C = \sum_{i=1}^{m} S_i \cdot n_i \cdot (1-b_i p_i) \quad (15)$$

Similarly to the proposed approach of building progressive tax scale, i.e., taking into account the condition of the linear increase in the tax rate:

$$n_i = n_i + \Delta(i-1)$$

$$\Delta = n_{i+1} - n_{i} = const \quad (16)$$

Let us build a progressive scale with a linearly increasing tax rate.

Also demanding that progressive scale built so should provide the same amount of taxes collected, as flat.

$$C_0 = n_0 \sum_{i=1}^{m} S_i = n_0 \cdot S_0 \quad , \quad (17)$$

i.e., $C = C_0 \quad (18)$

Formulas (14), (16) and (17) give us:

$$S_1 n_1 (1-b_1 p_1) + S_2 n_2 (1-b_2 p_2) + \ldots + S_m n_m (1-b_m p_m) - S_0 n_0 = 0 \quad (19)$$

Applying (15-18), we have:

$$S_1 n_1 (1-b_1 p_1) + S_2 \left( n_1 + \Delta \right) (1-b_2 (n_1 + \Delta)) + S_3 \left( n_1 + 2\Delta \right) (1-b_3 (n_1 + 2\Delta)) + \ldots + S_m \left( n_1 + (m-1)\Delta \right) (1-b_m (n_1 + (m-1)\Delta)) - S_0 n_0 = 0$$

or

$$S_1 n_1 (1-b_1 p_1) + S_2 \left( n_1 + \Delta \right) (1-b_2 (n_1 + \Delta)) + S_3 \left( n_1 + 2\Delta \right) (1-b_3 (n_1 + 2\Delta)) + \ldots + S_m \left( n_1 + (m-1)\Delta \right) (1-b_m (n_1 + (m-1)\Delta)) - S_0 n_0 = 0$$

$$+S_m \left( n_1 + (m-1)\Delta \right) (1-b_m (n_1 + (m-1)\Delta)) - S_0 n_0 = 0 \quad (20)$$
$n_{i} \left( S_{i} + S_{2} + S_{3} + \ldots + S_{m} \right) - n_{0} S_{0}$
$+ \Delta \left( S_{2} + 2S_{3} + \ldots + (m-1)S_{m} \right)$
$- n_{i}^{2} \left( S_{i} b_{1} + S_{2} b_{2} + S_{3} b_{3} + \ldots + S_{m} b_{m} \right)$
$- 2\Delta n_{i} \left( S_{i} b_{2} + 2S_{i} b_{3} + \ldots + (m-1)S_{m} b_{m} \right)$
$- \Delta^{2} \left( S_{i} b_{2} + 4S_{i} b_{3} + \ldots + (m-1)^{2}S_{m} b_{m} \right) = 0$

Finally, this equation can be described as:

$$\alpha \Delta^{2} + \beta \Delta + \gamma = 0 \quad \text{(22)}$$

Where,

$$\alpha = \sum_{j=1}^{m} (j-1)^{2} S_{j} b_{j}$$

$$\beta = \sum_{j=1}^{m} (j-1) S_{j} - 2n_{i} \sum_{j=1}^{m} (j-1) S_{j} b_{j}$$

$$\gamma = (n_{i} - n_{0}) S_{0} - n_{i}^{2} \sum_{j=1}^{m} S_{j} b_{j}$$

From Equation (21) we find the growth rate of the tax group $\Delta$:

$$\Delta_{l,2} = -\beta \pm \sqrt{\beta^{2} - 4\alpha \gamma} \quad \frac{2\alpha}{2\alpha} \quad \text{(24)}$$

Since $\Delta$ - Purely positive value, the negative root in (23) should be dropped:

$$\Delta = -\beta + \sqrt{\beta^{2} - 4\alpha \gamma} \quad \frac{2\alpha}{2\alpha} \quad \text{(25)}$$

Substituting the values found in the $\Delta$ (15), we obtain the final formula for finding $n_{i}$:

$$n_{i} = n_{i} + \Delta \cdot (i-1), \quad i = 1, 2, \ldots, m \quad \text{(26)}$$

Thus, in the general case, we obtain (22), (24) and (25) to build a progressive tax scale, providing the same tax levies on personal income as the flat rate, provided based on the probability of non-payment of tax, depending on the increase tax rates.

Hereafter it is more comfortable in the Formula (22) to move to dimensionless quantities $\eta_{j}$:

$$\alpha = S_{0} \sum_{j=1}^{m} (j-1)^{2} \eta_{j} b_{j}$$

$$\beta = S_{0} \left[ \sum_{j=1}^{m} (j-1) \eta_{j} - 2n_{i} \sum_{j=1}^{m} (j-1) \eta_{j} b_{j} \right] \quad \text{(27)}$$

$$\gamma = S_{0} \left[ n_{i} - n_{0} - n_{i}^{2} \sum_{j=1}^{m} \eta_{j} b_{j} \right]$$

Where $\eta_{j} = \frac{S_{j}}{S_{0}}$.

Considering the special case where in the Formula (9) all $b_{j} = 1$, i.e., $p_{j}=1-n_{i}$ Formula (26) take the form:

$$\alpha = S_{0} \sum_{j=1}^{m} (j-1)^{2} \eta_{j}$$

$$\beta = S_{0} \left( 1 - 2n_{i} \right) \sum_{j=1}^{m} (j-1) \eta_{j} \quad \text{(28)}$$

$$\gamma = S_{0} \left[ n_{i} - n_{0} - n_{i}^{2} \sum_{j=1}^{m} \eta_{j} \right]$$

### 3.3. Construction of the Linear Progressive Tax Scale for the Proposed Model

To test the applicability of the proposed probabilistic model of a progressive income tax based on actual statistical data it is crucial to construct tax scale with and without evasion of income tax and comparable results.

#### 3.3.1. Construction of the linear progressive tax scales, excluding tax evasion

On the basis of statistical data of the Federal State Statistics Service of the Russian Federation for 2014 (to find the value of income distribution $\eta$) it is needed to build a progressive tax scale by the method of linear transformation. At the same time, considering the case in which the tax reform is not accompanied by an order to increase the total income tax, i.e., at equality of the tax base and the amount of total income tax before and after the introduction of a progressive tax scale:

$$S = \sum_{i=1}^{m} S_{i} = S_{0} \quad \text{and} \quad C = \sum_{i=1}^{m} \eta_{i} \cdot S_{i} = C_{0} \quad \text{(29)}$$

The results of calculation of tax rates by the Formula (1) in the case of constant tax yield are presented in Table 1 (Section I).

The range of low-income tax group $n_{i}$ varies from 10% to 0%. Notable, that many countries use full exemption from tax for low-income groups, so the consideration of options $n_{i}=0$ is important.

The Table 1 (Section I) can be seen as a redistribution of the tax load on low-income groups to the wealthy. It is worth noting, that

<table>
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<td></td>
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<td>5</td>
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<td>10.4</td>
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<td>15.8</td>
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<td>0</td>
<td>4.4</td>
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<td>13.9</td>
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<td>10</td>
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<td>11.9</td>
<td>13.8</td>
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<td>17.5</td>
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<td>5</td>
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<td>12.1</td>
<td>15.7</td>
<td>19.3</td>
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<td>5.2</td>
<td>10.5</td>
<td>15.7</td>
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when the load for the fourth group of taxpayers has not changed, for the fifth it increased by 35%.

Further we will consider the case where the transformation of a flat tax should be accompanied by an increase in the total income tax.

For this we introduce the coefficient of the planned increase in tax yield due to the increase of the total income tax in \( \tau \) times: \( C = \tau \cdot C_0 \).

In the situation of flat rate taxation additional tax burden will fall on the entire population; in the progressive linear scale tax rates will be calculated for the tax base \( \tau \cdot C_0 \) with the formula:

\[
n_\tau = n_0 + \frac{(\tau \cdot n_0 - n_i)(i-1)}{\sum_{j=1}^{m}(j-1)n_i}
\]

(30)

The results of calculation of tax rates by the Formula (29) needed to increase tax yield by 20\%, are shown in Table 1 (Section II). Gradations of tax rates \( n_\tau \) and the distribution of income of the population by groups \( n_i \) are similar to the data contained in Table 1 and Section I.

The results, shown in Table 1 (Sections I and II), let us visually compare the tax rates with an increase in tax yields of the total income tax by 20\%.

### 3.3.2. Construction of the linear progressive tax scales taking into account the tax evasion

On the basis of the same statistical data of the Federal State Statistics Service of the Russian Federation for 2014, according to the currently existing income distribution \( n_i \), we construct a linear progressive tax scale, taking into account the possibility of tax evasion, while maintaining revenues in the case of switching to the progressive taxation.

Substituting these data into the Formula (27), we obtain:

\[
\alpha = S_0 \cdot 10.34,
\beta = S_0 (1 - 2n_i) \cdot 2.976,
\gamma = S_0 \left( n_i - n_0 - n_i^2 \right)
\]

After setting these expressions in (24) we can obtain the desired value of \( \Delta \) for different values of \( n_i \).

The results of these calculations for the three values of \( n_1 = 0.1; n_i = 0.05 \), and \( n_i = 0 \) are summarized in Table 2, Section I (wherein \( n_0 = 0.13 \)).

Thus, in accordance with the method of linear transformation were formed and constructed linear progressive tax scales, considering the possibility of evasion of personal income tax, while maintaining budget revenues compared to the flat tax rate.

### Table 2: The results of calculations of progressive linear scale adjusted tax rates, taking into account the probability of tax evasion, author’s calculations

<table>
<thead>
<tr>
<th>Section no</th>
<th>( n_j ) (%)</th>
<th>( i=1 )</th>
<th>( i=2 )</th>
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<th>( i=4 )</th>
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<td>7.8</td>
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<td>17.5</td>
<td>20.0</td>
<td></td>
</tr>
<tr>
<td>(( \tau = 1.2 ))</td>
<td>5</td>
<td>8.5</td>
<td>12.0</td>
<td>15.5</td>
<td>19.0</td>
<td></td>
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<tr>
<td></td>
<td>0</td>
<td>4.5</td>
<td>9.0</td>
<td>13.5</td>
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</table>

Now let us consider the case of evasion of income tax, with the possible increase in total budget revenues from paying it by introducing a progressive tax scale.

If we want to collect tax yields in \( \tau \) times greater than the flat rate tax, then:

\[
C = \tau \cdot n_j \sum_{j=1}^{m} S_j = n_0(\tau) \sum_{j=1}^{m} S_j
\]

(31)

Based on the foregoing, the problem solution reduces to the previously considered, providing that in mathematical expressions and final formulas we replace \( n_j \) for \( n_\tau = \tau \cdot n_0 \).

The results of adjusted tax rates calculations, taking into account the probability of avoiding tax payment obtained by linear transformation, provided the planned increase in tax yields from personal income taxes are presented in Table 2 (Section II).

### 4. CONCLUSION

Based on the results shown in Table 2, and comparing them with the results of the calculations presented in Table 1, we can draw the following conclusions:

- It is possible to build a progressive taxation scale, taking into account the probability of tax evasion, or ensuring equality of tax yield compared to a flat scale (Table 2 and Section I), or its increase in \( \tau \) time (Table 2 and Section II);
- An increase in tax yields from personal income taxes as a result of the progressive tax scale, which takes into account the likelihood of tax evasion, is achieved due to some increase in tax rates: In particular, with \( n_i = 10\% \) increase in tax rates for the second to fifth groups of the population amounted to 0.6-2.4\%; when \( n_i = 5\% 0.1-0.4\% \);
- The likelihood of non-payment of tax leads to a slight increase in tax rates, but tax levies are not reduced.

These findings and the results obtained for the linear progressive scale. Similarly, we can obtain the corresponding results for the non-linear taxation scale.
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