Management of the Formation of Rating Preferences of Economic Entities upon Collective Choice

Yury D. Motorygin1*, Vladimir S. Artamonov2, Alexander V. Maximov3, Elena N. Trofimets4, Valeriy Y. Trofimets5

1Saint Petersburg University of State Fire Service of EMERCOM of Russia, 149, Moskovsky Avenue, Saint Petersburg, 196105, Russia, 2Russian Ministry for Civil Defense, Emergencies and Elimination of Consequences of Natural Disasters, Teatralny prospect, 3., Moscow, 109012, Russia, 3Saint Petersburg University of State Fire Service of EMERCOM of Russia, 149, Moskovsky Avenue, Saint Petersburg, 196105, Russia, 4Saint Petersburg University of State Fire Service of EMERCOM of Russia, 149, Moskovsky Avenue, Saint Petersburg, 196105, Russia, 5Saint Petersburg University of State Fire Service of EMERCOM of Russia, 149, Moskovsky Avenue, Saint Petersburg, 196105, Russia. *Email: malygin_com@mail.ru

ABSTRACT

The applicability of a set of methods in the case of obtaining mixed results of the final ranking of economic entities in the process of their rating assessment has been substantiated. A new method of the formation of rating preferences upon collective choice has been proposed. The methodology is based on the following: An information base in the form of a summary table of ratings of compared entities; a procedure for the formation of rating preferences based on a rank-sum (or arithmetic means of ranks); a procedure for the formation of rating preferences with the joint application of rating arithmetic means and position indicators; a procedure for the formation of rating preferences using the Kemeny median. The proposals for the calculation of weighting factors of the voting procedures and modification of the procedure for profiling the preferences using the Kemeny median have been made. An algorithm for the use of the proposed method has been developed. Methodology testing has been conducted upon the rating assessment of the financial and economic condition of industrial enterprises.

Keywords: System Analysis, Management of Economic Systems, Economic-mathematical Modeling, Rating Assessment, Financial and Economic Condition

JEL Classifications: P40, P49

1. INTRODUCTION

The lack of information on business entities that complicates the process of taking managerial decisions by the business management regarding the selection of partners in the course of their activities is one of the urgent problems of the modern economy. The lack of information makes analysts look for new ways of getting the information on specific business entities.

The formation of ratings, which is a comparison of economic entities by a range of qualitative and quantitative characteristics and their ranking in order to identify the best and worst ones, is one of the ways to disseminate information on economic entities available to all interested users. Thus, currently much attention is paid to the calculation of various ratings of different economic entities, starting with the countries and ending with individual enterprises.

A number of major specialized rating agencies operate in the world economy. They provide sufficiently high quality services for the calculation of ratings. The services of these agencies are very popular because of high utility of the information provided. The features of the national economy often complicate the calculation of ratings with the use of foreign methods. This resulted in the emergence of a large number of original methods, the authors of which tried to take into account the peculiarities of the activities of various economic entities. Nevertheless, the problem of constructing adequate methods for their ranking is still relevant, including in terms of methodology: Data bases of methods, weight coefficients of local criteria, methods of obtaining the integrated
rating score and a number of other elements differ. Therefore, the use of different methods often leads to controversial conclusions about the final ranking of economic entities.

The aggregation of results obtained under different procedures is one of the approaches to improve the adequacy of the final ranking. Certainly, this approach leads to a significant increase in calculation costs, but, taking into account the modern development of computer technology, this aspect is of secondary importance for the problem under review.

2. LITERATURE REVIEW

The problem of the formation of ratings of economic entities has different aspects-practical, methodological, financial and economic, mathematical aspects-which was reflected in numerous publications on this theme. An overview of approaches to the formation of rating systems by Russian and Western agencies can be found in the works (Karminskiy et al., 2005; Karminskiy and Polozov, 2016). The banking sphere was an initial direction of ratings. A review of approaches to the construction of bank rating models, including as a basis for the systems of early warning of defaults is presented in the works (Altman and Saunders, 1998; Amato and Furline, 2004; Trueck and Rachev, 2008; Yuksel, 2010; Rognoni, 2011; Karminskiy, 2015). The article (Altman and Rijken, 2004) is one of the fundamental works in the sphere of rating models, which examines an issue of the stability of such models.

Currently, corporate ratings are gaining a growing importance. With regard to industrial enterprises, they play an important role in the organization of syndicated lending and public tenders and serve as a basis for the formation of corporate bond ratings (Carling et al., 2007). A large number of works are devoted to the issues of the rating assessment of the financial status of enterprises (Kotlyar, 1999; Schiborsch, 2000; Postyuşkov, 2001; Kovalev and Volkova, 2002; Ginzburg, 2004, etc.).

The mathematical aspects of rating procedures are reflected in the numerous works on econometrics, mathematical statistics, quantitative methods of financial analysis, expert evaluations (Podinovskiy, 2007; Goryunov, 2012; Saaty and Vargas, 2013; Eiselt and Sandblom, 2013; Doumpos and Zopounidis, 2014; Batkovskiy, 2015, etc.).

The mathematical aspects of the rating procedures are reflected in numerous works on econometrics, mathematical statistics, quantitative financial analysis methods, expert evaluations (Podinovskiy, 2007; Goryunov, 2012; Saaty and Vargas, 2013; Eiselt and Sandblom, 2013; Doumpos and Zopounidis, 2014; Batkovskiy, 2015, etc.).

The integral use of economic-mathematical methods and models is one of the principles of their application in the solution of complex management problems. For this purpose, two directions - the parallel use of methods (simultaneous use of different methods at the same stage of the problem solution) and sequential use of methods (the use of different methods at different stages of the problem solution) - can be distinguished.

The need for the parallel use of different methods (models) is due to the need to improve (enhance) the solution, which is obtained by applying a single method (model). A set of models jointly used to solve the same problem, is called an ensemble (committee) of models. In the last decade, ensembles of models have become an area of active research in data mining (Nisbet et al., 2009; Perner, 2010, Alvo and Yu, 2014; Larose and Larose, 2015, etc.).

3. METHODOLOGY

3.1. Methodology Information Base

A concept of a rank of an entity underlies the proposed methodology of forming the rating preferences of economic entities under collective choice.

A rank ($r$) of an entity shall mean a serial number of the entity in the list of analyzed entities after their ordering upon descending ranking values. In this case, the rule of rank assignment to the entities shall be as follows:

1. If in a set of rating values all numbers are different, each entity is given a unique rank $r_i$;
2. If in a set of rating values there is a group of $k$ equal ranking values $x_{i1} = x_{i2} = ... = x_{ik}$, then the rank of the respective entities will be the same and equal to the arithmetic means of their serial numbers. The entity following this group gets a rank equal to $r_i + k$;
3. The entity with the maximum rating value shall have a rank equal to $r_i$;
4. The entity with the minimum rating value has the highest rank value equal to $n - k_{min} + 1$, where $n$ is a number of entities in the list, $k_{min}$ means a number of repeating minimum rating values in the list of entities.

Using the introduced concept of a rank of an entity, a profile of entities preferences (entities ranking) is developed for each of the aggregated methods of the rating assessment. Preference profiles are grouped in a summary table of entities ranks (Table 1), which is an information base for the methodology of the formation of rating preferences under collective choice.

3.2. Formation of the Rating Preferences Based upon the Rank Sum or Ranking Arithmetic Means

Final ranking of entities can be constructed based on the data of Table 1 by calculating the sum of ranks for each of the entities analyzed ($R_i = \sum_{j=1}^{n} r_{ij}$) and their subsequent ordering in ascending order $R$. This method is called a rank sum method. It is obvious that ranking will be similar if we divide each of the

<table>
<thead>
<tr>
<th>Methods</th>
<th>Compared entities</th>
<th>Entity 1</th>
<th>Entity 2</th>
<th>...</th>
<th>Entity j</th>
<th>...</th>
<th>Entity n</th>
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<tr>
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<td>$r_{12}$</td>
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<td>$r_{1j}$</td>
<td>...</td>
<td>$r_{1n}$</td>
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<td>$r_{22}$</td>
<td>...</td>
<td>$r_{2j}$</td>
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<tr>
<td>Method i</td>
<td>$r_i$</td>
<td>$r_{ij}$</td>
<td>...</td>
<td>$r_{ij}$</td>
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<td>$r_{im}$</td>
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<td>...</td>
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<td>...</td>
<td></td>
</tr>
<tr>
<td>Method m</td>
<td>$r_{m1}$</td>
<td>$r_{m2}$</td>
<td>...</td>
<td>$r_{mj}$</td>
<td>...</td>
<td>$r_{mn}$</td>
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</tbody>
</table>
values $R_i$ by the number of the methods used $m$. This method is called a method of ranking arithmetic means.

The main advantage of the rank sum method and the method of ranking arithmetic means is in their simplicity. However, it should be noted that they have a significant disadvantage. This disadvantage is caused by the fact that the ranks are measured on an ordinal scale. The works (Schrader, 1971; Kemeny and Snell, 1972; Orlov, 1996) show that only members of the variation series, in particular the median, can be reasonably used as an arithmetic mean of all the mean values in an ordinal scale. However, in practice, it is impractical to completely ignore the arithmetic mean value, due to the familiarity, clarity and, as a consequence, its high prevalence. However, when using the arithmetic mean there occur well-known situations known as “the average temperature in the hospital” or “the central part of the donut.” Such contradiction can be resolved if we take into account the fact that the arithmetic mean is a sufficiently stable score at a small scatter of data being processed, otherwise its use is a purely formal procedure. Thus, it can be concluded that the final ranking upon the rank sum method or the method of ranking arithmetic means should be carried out only with adequate consistency of preference profiles obtained by different methods.

A concordance coefficient can be used to assess the degree of consistency of preference profiles. This coefficient is defined by the following formula:

$$W = \frac{12S}{m^2n(n^2-1)-m\sum T_i}$$

Where $S$ - the sum of squares of the differences between the sum of ranks and their average value; $m$ - the number of the methods used; $n$ - the number of the entities analyzed; $T_i$ - an indicator of linked ranks in $i$th preference profile.

If a summary table of entities is used as the information base, a value $S$ can be determined by the following formula:

$$S = \sum_{i=1}^{n} \left[ \sum_{j=1}^{m} r_{ij}^2 \frac{1}{2} m(n+1) \right]$$

Where $r_{ij}$ - a rank assigned to the $j$th entity upon the $i$th method.

An indicator of linked ranks $T_i$ is calculated by the formula:

$$T_i = \sum_{k=1}^{m} (h_{ik}^j - h_{ik}) , i = 1 \ldots m$$

Where $p_i$ - the number of groups of equal ranks in the $i$th preference profile; $h_{ik}$ the a number of equal ranks in the $k$th group of linked ranks of the $i$th preference profile.

If there are no matching ranks in the preference profile, then $h_{ik} = 0$ and $p_i = 0$, consequently $T_i = 0$.

The analysis of the national and foreign literature allows to set the limit value for the concordance coefficient $W = 0.5$ (Schrader, 1971; Kemeny and Snell, 1972). When $W < 0.5$ preference profiles are not consistent, when $0.5 \leq W < 0.7$ - not sufficiently consistent, when $0.7 \leq W < 0.9$ - consistent, when $W \geq 0.9$ - extremely consistent.

When $W \geq 0.7$ preference profiles are considered consistent and in order to develop the final ranking of enterprises we can use the rank sum method or the method of ranking arithmetic means. When $W < 0.7$ preferences profiles are not sufficiently consistent and to develop the final ranking the use of the mentioned methods is not entirely correct, which is particularly manifested when $W < 0.5$. Therefore, there is a need to develop other approaches and methods that make it possible to overcome the disadvantages of the rank sum method or the method of ranking arithmetic means.

### 3.3. Formation of Rating Preferences upon the Joint Application of the Method of Ranking Arithmetic Means and the Method of Ranking Position Indicators

The proposed approach is designed to compensate in some way for the shortcomings of the rank sum method and the method of ranking arithmetic means and provides more correct development of the final ranking of the entities at a value of concordance coefficient $W < 0.7$.

The approach implies the joint application of the method of ranking arithmetic means and the method of ranking position indicators. This approach is consistent with the concept of sustainability, considering the use of a variety of methods for processing the same data in order to form conclusions, obtained simultaneously in all methods used.

To adjust the rankings obtained by the method of ranking arithmetic means, it is proposed to use the ranking position indicators. This approach is due to the fact that when measured on an ordinal scale (the rank scale is their representative) it is correct to use the members of the variation series (in particular, a median should be used as a mid-scale, which can be mathematically proved). Quartiles, deciles, percentiles, etc. can be the members of the variation series, describing the position of the data.

It is proposed to use three position indicators: A median, a lower quartile and an upper quartile.

A preference profile median shall mean the importance of the rank located in the middle of the profile. For a profile with an odd number of ranks, a rank located in the center of the profile will be a median. For a profile with an even number of ranks, an arithmetic mean of two central ranks will be a median.

Preference profile quartiles shall mean the significance of the ranks, dividing the preference profile into four equal parts. The lower quartile separates the one-fourth of the profile with the lowest values of ranks, while the upper quartile separates the one-fourth of the profile with the highest values of ranks. Respectively, a median is a medium quartile.
The approach implies the element-wise summation of the final ranks obtained by the method of arithmetic means and the method of ranking position indicators:

$$R_j^{AP} = R_j^{A} + R_j^{P}$$

(4)

Where $R_j^{A}$ - a final rank of the jth entity obtained by the method of ranking arithmetic means; $R_j^{P}$ - a final rank of the jth entity obtained by the method of ranking position indicators; $R_j^{AP}$ - a final rank of the jth entity obtained upon the joint application of the method of arithmetic means and the method of ranking position indicators.

The final ranking is the result of the ordering of final ranks $R_j^{AP}$ in ascending order.

3.4. Formation of Rating Preferences Based on the Kemeny Median

With the formation of rating preferences, the natural presumption is that the final ranking should be as close as possible to the preference profiles (rankings) based on different methods. Such ranking will correspond to the Kemeny median:

$$P^{*} = \arg \min_{P} \sum_{j=1}^{n} d(P_A, P_B)$$

(5)

Where $d(P_A, P_B)$ - the distance between rankings constructed by using the methods A and B.

The construction of the final ranking with the use of the Kemeny median requires the introduction of the distance between any two rankings $P_A$ and $P_B$. In the basic procedure for finding the Kemeny median, such distance is introduced on the basis of the matrix of relations and the matrix of losses, the construction and processing of which require a sufficiently large number of additional processing operations.

The specifics of the problem considered allows to avoid the construction of these matrices, since the ranks of entities derived from the calculated rating scores by the methods described above, not the expert opinions located in the space of non-numeric objects, serve as the processed data. Taking into account the specifics of the problem, a concept of distance between the rankings should be introduced.

Definition: The distance between the rankings $P_A$ and $P_B$, described by the vectors $\|P_A^j\|$ and $\|P_B^j\|$ respectively, is a value $d(P_A, P_B)$ equal to the sum of modules of differences of the elements located on the same places in their respective vectors, i.e., the distance between the rankings $P_A$ and $P_B$ and is calculated as follows:

$$d(P_A, P_B) = \sum_{j=1}^{m} |P_A^j - P_B^j|$$

(6)

Based on the introduced concept of distance between the rankings, the problem of finding the final ranking can be expressed in the following optimization formula:

$$\sum_{i=1}^{m} d(P_i, P^{*}) \min$$

(7)

Where $P^{*}$ - the final ranking.

The final ranking $P^{*}$ can be considered as a set element $(P_i)$. In this case, the solution of the optimization problem will be reduced to one of the rankings that are based on the above methods. In our opinion, it is more reasonable to find a new ranking $P_{\text{new}} = P^{*}$, the sum of distances from which to all other rankings is minimal. At the same time, a danger of a situation of “getting into the centre of the donut” is smoothed over by the non-Euclidean nature of the introduced metrics and optimization nature of the task. Consequently, the final ranking $P^{*}$ obtained based on the modified Kemeny median is more objective in comparison with the ranking obtained based on the rank sum method.

3.5. The Algorithm of the Formation of Rating Preferences upon Collective Choice

Having considered the specific issues associated with the development of the method of the formation of rating preferences of economic entities upon collective choice, we propose an algorithm for the use of the developed method as a summary logical conclusion (Figure 1).

4. RESULTS

4.1. A Brief Description of the Task of the Rating Assessment of the Financial Status of Enterprises

Testing of the proposed methodology has been carried out on the task of the rating assessment of the financial status of industrial enterprises.

The financial status of an enterprise is a complex characteristic of its position, which is determined by a system of indicators reflecting the availability, distribution and use of financial resources of an enterprise. The rating assessment of the financial status is a tool for comparing the indicators of financial and economic activities of competing companies. As a rule, it is conducted in the interests of external users: Investors, shareholders, creditors, customers, tax authorities, auditors, etc.

The rating assessment is based on the official financial statements containing the limited information on the activities of the enterprise. It does not imply an in-depth study of various aspects of financial-economic activities of the enterprise and its main purpose is to get an integrated rating numeric score for the subsequent ranking of compared enterprises.

Today in Russia, there are different methods of the rating assessment of the financial status of enterprises. The conducted analysis of the subject area and literary sources of educational, scientific and applied nature has allowed to identify a number of methods, which are, in our opinion, the most widely used. Such methods are as follows: A method of Bakanov and Sheremet (Bakanov and Sheremet, 2001) a method of Ginzburg (Ginzburg, 2004), a method of Grafova (Grafova, 2003), a method of the...

However, it should be noted that in a number of methods there is a clearly defined system of indicators, based on which the rating score is calculated. There are only general guidelines for its construction. This circumstance does not allow for the correct comparative analysis of these methods. Therefore, for further consideration we have selected methods, the authors of which, in addition to the method of the rating score calculation, provide a system of indicators, which is an information base for the rating calculation. These methods include the methods of Kovalev, the company INEC, Schiborsch, Ginzburg, Postyushkov.

To test the developed method, six industrial companies were selected (with the code names “Quantum,” “Prometheus,” “Emerald,” “Pulsar,” “Alpha” and “Spectrum”), which provided the necessary reports for the calculation of indicators of their financial status.

**Figure 1:** The algorithm of the formation of rating preferences upon collective choice

```
Start

Method 1  Method 2  Method k  Method m

Formation of the summary table of entities ranks

Calculation of the concordance coefficient \( W \)

no

yes

\( W < 0.7 \)

Formation of the collective rating score upon the arithmetic means and position indicators

Is there the Kemeny implementation?

no

yes

Formation of the collective rating score upon the Kemeny median

Formation of the collective rating score by two rankings upon the rank sum

The end
```
4.2. Rating Assessment of the Financial Status of Enterprises

In accordance with the algorithm of the application of the developed method, the rating assessment of enterprises by using the methods developed by Kovalev, INEK company, Schiborsch, Ginzburg and Postyushkov has been conducted at the first stage. The calculated final rating values are given in Table 2.

A summary table of ranks of enterprises is formed on the basis of Table 2 and the rules of rank assignment (Table 3).

At the next stage, the assessment of the degree of consistency of preference profiles is conducted. For this purpose the concordance coefficient is calculated by the formula (1):

\[ W = \frac{12S}{m^2n(n^2-1)-m\sum T_i} = \frac{12 \times 256.5}{5^2 \times 6 \times (6^2-1)-5 \times 12} = 0.593 \]

Since \(0.5 \leq W < 0.7\), the preference profiles are deemed not sufficiently consistent. Thus, to construct the final ranking it makes most sense to use either a method of the collective rating formation based on the joint use of arithmetic means and ranking position indicators, or a method of the collective rating formation based on the modified Kemeny median.

Let us conduct the rating assessment of the selected enterprises, using the above methods.

Using the first method, we calculate the final ranks of the enterprises by using arithmetic means of ranks, based on ranking position indicators and on their joint application (Table 4).

The final ranking is the result of the final rank ordering \( R_{\text{AP}} \) in ascending order (Table 5).

The second method that has been proposed for the formation of rating preferences is based on the application of the Kemeny median.

The first phase of this method involves the construction of the matrix of distances between the rankings obtained by using methods of Kovalev, INEC company, Schiborsch, Ginzburg and Postyushkov upon the formula (6) (Table 6).

After the construction of the matrix of distances between the final rankings, we obtain the final ranking \( P^* \) based on the solution of the optimization task (7).

The task (7) has a significant computational complexity, so for its solution it is advisable to use the appropriate software. The Excel add-in “Search for Solution” is recommended as such software. The Kemeny median found is given in Table 7.

The final ranking of enterprises \( P^* \) is shown in Table 8.

5. DISCUSSION

Analyzing the results obtained (Tables 5 and 8), it is easy to note that the final rankings obtained on the basis of the two proposed methods are slightly different from one another. This applies to

<table>
<thead>
<tr>
<th>Method</th>
<th>Enterprises</th>
<th>Quantum</th>
<th>Prometheus</th>
<th>Emerald</th>
<th>Pulsar</th>
<th>Alpha</th>
<th>Spectrum</th>
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<tr>
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<td>15.0</td>
<td>49.5</td>
<td>42.6</td>
<td>68.0</td>
<td>23.3</td>
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<tr>
<td>INEK</td>
<td>22.0</td>
<td>22.0</td>
<td>27.0</td>
<td>8.0</td>
<td>14.0</td>
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<td></td>
</tr>
<tr>
<td>Schiborsch’s</td>
<td>−0.5</td>
<td>7.6</td>
<td>10.9</td>
<td>11.6</td>
<td>−3.3</td>
<td>12.8</td>
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<td>288.0</td>
<td>192.0</td>
<td>167.0</td>
<td>167.0</td>
<td>266.0</td>
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<tr>
<td>Postushkov’s</td>
<td>1.456</td>
<td>1.219</td>
<td>1.046</td>
<td>−0.285</td>
<td>−0.788</td>
<td>2.649</td>
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<tr>
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<td>4.0</td>
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<th>Alpha</th>
<th>Spectrum</th>
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</thead>
<tbody>
<tr>
<td>Final ranking upon arithmetic means</td>
<td>5.0</td>
<td>3.5</td>
<td>2.0</td>
<td>3.5</td>
<td>6.0</td>
<td>1.0</td>
<td></td>
</tr>
<tr>
<td>Final ranking upon position indicators</td>
<td>5.0</td>
<td>4.0</td>
<td>2.5</td>
<td>2.5</td>
<td>6.0</td>
<td>1.0</td>
<td></td>
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<tr>
<td>Summary ranking upon arithmetic means and position indicators</td>
<td>10.0</td>
<td>7.5</td>
<td>4.5</td>
<td>6.0</td>
<td>12.0</td>
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<tr>
<td>Final ranking upon arithmetic means and position indicators</td>
<td>5.0</td>
<td>4.0</td>
<td>2.0</td>
<td>3.0</td>
<td>6.0</td>
<td>1.0</td>
<td></td>
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</table>
the enterprises “Emerald” and “Pulsar,” as well as enterprises “Quantum” and “Alpha,” which had the same position in the second ranking. If we take into account the results of the two final rankings, it can be assumed that the financial-economic condition of the enterprise “Emerald” slightly exceeds the financial-economic status of the enterprise “Pulsar,” while the financial-economic condition of the enterprise “Quantum” significantly exceeds the financial-economic condition of the enterprise “Alpha.”

Despite some differences in the final rankings, their concordance coefficient $W = 0.985$, indicating a high degree of consistency of the estimates, so the resulting ranking can be constructed by using the rank sum method (Table 9).

The obtained resulting ranking confirmed our assumptions about certain superiority of the enterprise “Emerald” to the enterprise “Pulsar,” and the enterprise “Quantum” - to the enterprise “Alpha.”

With the accumulation of statistical information on the rating assessment of enterprises, weight coefficients for each of the discussed methods can be introduced into the proposed procedure for finding the Kemeny median, which allows to increase even more the objectivity of the resulting rating score.

Obviously, if the ranking $P_A$ is closer to the final ranking $P^*$ than the ranking $P_B$, the weight of the method $A$ must be higher than the weight of the method $B$, and vice versa. Since the best ranking corresponds to a smaller distance, the weight coefficients can be found by the following formula:

$$ V_i = \frac{1}{\sum_{j=1}^{m} \frac{1}{d(P_i, P^*)}} $$

(8)

Where $V_i$ - a weight coefficient of the $i^{th}$ method.

Thus, for the above example the calculated weights of methods are shown in Table 10.

It should be noted that the weight coefficients calculated upon the results of the current procedure may be used in the subsequent procedures. It is obvious that if the other enterprises are involved in the rating assessment, then the weights calculated by the formula (8) will differ from the weights calculated last time. In order to stabilize the weights, the current and previously calculated weights are averaged, thus a new weight takes account of the value of “old” weights. This procedure is recurrent in nature and corresponds to the procedure for exponential smoothing of weights with a coefficient equal to $\alpha = 0.5$.

Thus, with regard to the weights of methods, the formula of finding the final ranking can be written as follows:

$$ \sum_{i=1}^{m} (V_i \times d(P_i, P^*)) \min $$

(9)

Note that, despite its advantages, the method of the formation of collective preferences based on the modified Kemeny median has one significant drawback - it is much more complicated in terms of the applied calculation procedures. In particular, in order to find the final ranking $P^*$ the algorithms based on the method of branches and borders or gradient search methods should be used, so the practical application of the method is associated with its implementation at the software level. In the absence of specific software the task (9) can be efficiently solved by using the Excel add-in “Search for Solution,” in which a method of generalized gradient and the evolutionary method are implemented to solve nonlinear tasks.

6. CONCLUSION

Currently, there are various methods of the formation of ratings of economic entities. Each of the methods is usually a set of

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Table 5: The final ranking of enterprises based on the joint application of arithmetic means and ranking position indicators

<table>
<thead>
<tr>
<th>Rank</th>
<th>Place</th>
<th>Enterprise</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1st</td>
<td>Spectrum</td>
</tr>
<tr>
<td>2</td>
<td>2nd</td>
<td>Emerald</td>
</tr>
<tr>
<td>3</td>
<td>3rd</td>
<td>Pulsar</td>
</tr>
<tr>
<td>4</td>
<td>4th</td>
<td>Prometheus</td>
</tr>
<tr>
<td>5</td>
<td>5th</td>
<td>Quantum</td>
</tr>
<tr>
<td>6</td>
<td>6th</td>
<td>Alpha</td>
</tr>
</tbody>
</table>

Table 6: Matrix of distances between the rankings

<table>
<thead>
<tr>
<th>Method</th>
<th>Kovalev’s</th>
<th>INEK</th>
<th>Schiborsch’s</th>
<th>Ginsburg’s</th>
<th>Postushkov’s</th>
</tr>
</thead>
<tbody>
<tr>
<td>Kovalev’s</td>
<td>0.0</td>
<td>11.0</td>
<td>4.0</td>
<td>3.0</td>
<td>8.0</td>
</tr>
<tr>
<td>INEK</td>
<td>11.0</td>
<td>0.0</td>
<td>10.0</td>
<td>9.0</td>
<td>8.0</td>
</tr>
<tr>
<td>Schiborsch’s</td>
<td>4.0</td>
<td>10.0</td>
<td>0.0</td>
<td>3.0</td>
<td>8.0</td>
</tr>
<tr>
<td>Ginsburg’s</td>
<td>3.0</td>
<td>9.0</td>
<td>3.0</td>
<td>0.0</td>
<td>10.0</td>
</tr>
<tr>
<td>Postushkov’s</td>
<td>8.0</td>
<td>8.0</td>
<td>8.0</td>
<td>10.0</td>
<td>0.0</td>
</tr>
</tbody>
</table>

Table 7: The Kemeny median of rankings of the enterprises

<table>
<thead>
<tr>
<th>Enterprises</th>
<th>Kovalev’s</th>
<th>INEK</th>
<th>Schiborsch’s</th>
<th>Ginsburg’s</th>
<th>Postushkov’s</th>
</tr>
</thead>
<tbody>
<tr>
<td>Kemeny median</td>
<td>5.0</td>
<td>4.0</td>
<td>3.0</td>
<td>3.0</td>
<td>5.0</td>
</tr>
<tr>
<td>Final ranking upon the Kemeny median</td>
<td>5.5</td>
<td>4.0</td>
<td>2.5</td>
<td>2.5</td>
<td>5.5</td>
</tr>
</tbody>
</table>
indicators of the financial-economic condition of a business entity, the distribution of weight coefficients on the local criteria and the procedure for their integration into the integral rating indicator. Therefore, there is a problem of choosing any one method and conducting on its basis the rating assessment of economic entities or the simultaneous use of several proven methods. In our opinion, the latter approach is more reliable, but in this case there is a need to form the collective rating by combining some rankings obtained with the use of each of the methods. This integration leads to a significant increase in calculation costs. Thus, the practical application of the proposed approach implies the development of the automation system of the formation of collective rating preferences.

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