

The Relationship between Interest Rate, Exchange Rate and Stock Price: A Wavelet Analysis

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ABSTRACT: This paper examines the multi-scale relationship between the interest rate, exchange rate and stock price using a wavelet transform. In particular, we apply the maximum overlap discrete wavelet transform (MODWT) to the interest rate, exchange rate and stock price in US over the period from January 1990 to December 2008 and using the definitions of wavelet variance, wavelet correlation and cross-correlations to analyze the association as well as the lead/lag relationship between these series at the different time scales. Our results show that the relationship between interest rate and exchange rate is not significantly different from zero at all scales. On the other hand, the relationship between interest rate returns and stock index returns is significantly different from zero only at the highest scales. The exchange rate returns and stock index returns have a bidirectional relationship in this period at longer horizons.

Keywords: Wavelet analysis, Interest rate, Stock price, Wavelet cross-correlation, Granger causality
JEL Classification: C02, C22

1. Introduction

The stock markets are becoming an integral part of the economies of many countries. With the introduction of free and open economic policies and advanced technologies, investors are finding easy access to stock markets around the world. The fact that stock market indices have become an indication of the health of the economy of a country indicates the importance of stock markets. This increasing importance of the stock market has motivated the formulation of many theories to describe the working of the stock markets.

One piece of information which is linked to the stock markets is interest rates and stock price fluctuations. In theory, the interest rates and the stock price have a negative correlation. This is because a rise in the interest rate reduces the present value of future dividend's income, which should depress stock prices. Conversely, low interest rates result in a lower opportunity cost of borrowing. Lower interest rates stimulate investments and economic activities, which would cause prices to rise. On the other hand, according to the parity conditions, the interest rates and the exchange prices should be related with a negative coefficient. Hence, we would expect a relationship between exchange price and stock price with a positive coefficient.

However, the empirical studies carried out in the various markets had revealed conflicting results on causality between stock prices and the above economic variables. Mok (1993), verified the causality of daily interest rate, exchange rate and stock prices in Hong Kong for the period from 1986 to 1991. The results indicate that the HIBOR (Hong Kong Inter Bank Offered Rate) and the price indices are independent series. As a further extension to the study the relationship between exchange rate and stock price was examined, the research concluded that those series are also independent.

Hashemzadeh and Taylor (1988) have found bi-directional causality present in regression models between money supply and stock returns using stock indexes to estimate market returns. As regards, the interest rates of the results are not as conclusive. The direction of causality seems to be mostly

running from interest's rate to stock price but not the other way. Solnik (1987) found a weak positive relation between real stock return differential and the changes in the real exchange rate, and that a real growth on the stock market has also a positive influence on the exchange rate.

In this paper, we examine the relationship between interest rate, exchange rate and stock price. We adopt the time-series technique based in wavelet analysis. We apply the wavelet cross-correlation between these series based upon the maximum overlap discrete wavelet transform MODWT [Percival, and Mofjeld (1997) and Daubechies (1992)] families of wavelets. The decomposition of a time series on a scale-by-scale basis can unveil the structure at deferent time horizons. For example, the wavelet transform produces an alternative, known as the wavelet variance to the periodogram. This variance decomposition may be easily generalized for multivariate time series. Standard time-domain measures of association for multivariate time series (e.g., cross-covariance and cross-correlation) may be defined using the coefficients from the application of the wavelet transform to each series, thus producing the wavelet cross-covariance and wavelet cross-correlation.

The remainder of the paper is organized as follows. The main properties of the wavelets and the analytical deference's with other filtering methods are dealt with in section 2, where we present the statistical properties of the wavelet variance, correlation and cross-correlation. In section 3, we describe the data used in the study and the basic statistics. Section 4 discusses the empirical finding. Finally, section 5 concludes and highlights some relevant remarks.

2. Wavelet Analysis

The series were filtered using wavelet analysis that is a relatively new (at least for economists) statistical tool that, roughly speaking, decomposes a given series in orthogonal components, as in the Fourier approach, but according to scale (time components) instead of frequencies. The comparison with the Fourier analysis is useful first because wavelets use a similar strategy: find some orthogonal objects (wavelets functions instead of sines and cosines) and use them to decompose the series. Second, since the Fourier analysis is a common tool in economics, it may be useful in understanding the methodology and also the interpretation of results. We have to stress the main deference between the two tools. Wavelet analysis does not need stationary assumption in order to decompose the series. This is because the Fourier approach decomposes in frequencies space that may be interpreted as events of time-period T (where T is the number of observations). Put differently, spectral decomposition methods perform a global analysis whereas, wavelets methods act locally in time and so do not need stationary cyclical components. Recently, to relax the stationary frequencies assumption a windowing Fourier decomposition that essentially use, for frequencies estimation, a time-period M (the window) event less than the number of observations T . The problem with this approach is the right choice of the window and, more important, its constancy over time. Coming back to wavelets and going into some mathematical detail we may note that there are two basic wavelet functions: the father wavelet ϕ and mother wavelets ψ such that:

$$\int_R \phi(t)dt = 1 \quad \text{and} \quad \int_R \psi(t) = 0 \quad (1)$$

Using wavelets, any function in $L^2(\mathbb{R})$ can be written as a linear combination of the type

$$f(t) = \sum_j \sum_k \alpha_{j,k} \phi_{j,k}(t) = \sum_k S_{J,k} \phi_{J,k}(t) + \sum_{j>J} \sum_k d_{j,k} \psi_{j,k}(t) \quad (2)$$

Where $S_{j,k} = \langle f(t), \phi_{j,k}(t) \rangle$ and $d_{j,k} = \langle f(t), \psi_{j,k}(t) \rangle$ are the wavelet coefficients and where $\phi_{j,k}, \psi_{j,k}$ the so-called scaling are and wavelet functions, respectively. The formal definition of the father wavelets is the function

$$\phi_{j,k}(t) = 2^{-j/2} \phi(2^{-j}t - k) \quad (3)$$

defined as non-zero over a finite time length support that corresponds to given mother wavelets

$$\psi_{j,k}(t) = 2^{-j/2} \psi(2^{-j}t - k) \quad (4)$$

The former integrates to 1 and reconstructs the longest time-scale component of the series (trend), while the latter integrates to 0 (similarly to sine and cosine) and is used to describe all deviations from trend. The mother wavelets play a role similar to sines and cosines in the Fourier decomposition.

2.1. The Discrete Wavelet Transform

Let $h_1 = (h_{1,0}, \dots, h_{1,L-1}, 0, \dots, 0)^T$ denote the wavelet filter coefficients of a Daubechies compactly supported wavelet for unit scale Daubechies (1992), zero padded to length N by defining $h_{1,l}=0$ for $l > L$. A wavelet filter must satisfy the following three basic properties:

$$\sum_{l=0}^{L-1} h_{1,l} = 0 \quad ; \quad \sum_{l=0}^{L-1} h_{1,l}^2 = 1 \quad (5)$$

$$\sum_{l=0}^{L-1} h_{1,l} h_{1,l+2n} = 0 \quad \text{for all non zero integers } n. \quad (6)$$

That is, a wavelet filter must sum to zero (have zero mean), must have unit energy, and must be orthogonal to its even shifts.

Let $g_1 = (g_{1,0}, \dots, g_{1,L-1}, 0, \dots, 0)^T$ be the zero padded scaling filter coefficients, defined via $g_{1,l} = (-1)^{l+1} h_{1,L-l-1}$ and let $X_0, \dots, X_{N-1}$ be a time series. For scales that $N \geq L_j$, where $L_j = (2^j - 1)(L - 1) + 1$, we can filter the time series using h_j to obtain the wavelet coefficients

$$W_{j,t} = 2^{j/2} \tilde{W}_{j,2^j(t+1)+1}, \quad \left\lceil (L-2) \left(1 - \frac{1}{2^j}\right) \right\rceil \leq t \leq \left\lfloor \frac{N}{2^j} - 1 \right\rfloor, \quad (7)$$

Where,

$$\tilde{W}_{j,t} = \frac{1}{2^{j/2}} \sum_{l=0}^{L_j-1} h_{j,l} X_{t-1}, \quad t = L_j - 1, \dots, N - 1 \quad (8)$$

The $\tilde{W}_{j,t}$ coefficients are associated with changes on a scale of length $\tau_j = 2^{j-1}$ and are obtained by sub sampling every 2^j th of the $\tilde{W}_{j,t}$ coefficients, which forms a portion of the maximal overlap discrete wavelet.

However the orthonormal discrete wavelet transform (DWT), even if widely applied to time series analysis in many disciplines, has two main drawbacks: the dyadic length requirement (i.e. a sample size divisible by 2^j), and the fact that the wavelet and scaling coefficients are not shift invariant due to their sensitivity to circular shifts because of the decimation operation.

An alternative to DWT is represented by a non-orthogonal variant of DWT: the maximal overlap DWT (MODWT).

2.2. Maximal Overlap DWT (MODWT)

In contrast to the DWT, the MODWT Percival et al (2000) does not decimate the coefficients and therefore the number of scaling and wavelet coefficients at every level of the transform is the same as the number of sample observations. For this reason the MODWT is also called non-decimated DWT. Although it loses orthogonality and efficiency in computation, this transform does not have any restriction on the sample size and it is shift invariant. Wavelet coefficients, i.e. $\tilde{w}_{j,t}$ and scaling coefficients, i.e. $\tilde{v}_{j,t}$ at levels j ; $j = 1, \dots, J$, are obtained as follows:

$$\tilde{w}_{j,t} = \sum_{l=0}^{L-1} \tilde{g}_l \tilde{v}_{j-1,t-l \bmod N} \quad \text{and} \quad \tilde{v}_{j,t} = \sum_{l=0}^{L-1} \tilde{h}_l \tilde{v}_{j-1,t-l \bmod N} \quad (9)$$

The wavelet and scaling filters, $\tilde{g}_l, \tilde{h}_l$ are rescaled as $\tilde{g}_j = g_j / 2^{j/2}, \tilde{h}_j = h_j / 2^{j/2}$. Non-decimated wavelet coefficients represent differences between generalized averages of the data on a scale $\tau_j = 2^{j-1}$ (or level j).

MODWT provides the usual functions of the DWT, such as multiresolution decomposition analysis and cross-correlation analysis based on wavelet transform coefficients. But unlike the classical DWT, it can handle any sample size, it is translation invariant, as a shift in the signal does not change the pattern of wavelet transform coefficients; and provides increased resolution at coarser scales. In addition, MODWT provides a larger sample size in the wavelet correlation analysis and produces a more asymptotically efficient wavelet covariance estimator than the DWT.

2.3. The wavelet variance, covariance, correlation and cross-correlation

The basic idea of the wavelet variance is to substitute the notion of variability over certain scales for the global measure of variability estimated by sample variance.

The wavelet variance of stochastic process X is estimated using the MODWT coefficients for scale $\tau_j = 2^{j-1}$ through:

$$\hat{\sigma}_X^2(\tau_j) = \frac{1}{\hat{N}_j} \sum_{k=L_j-1}^{N-1} (\hat{W}_{j,k})^2 \quad (10)$$

where $\hat{W}_{j,k}$ is the MODWT wavelet coefficient of variable X at scale τ_j . $\hat{N}_j = N - L_j + 1$ is the number of coefficients unaffected by boundary, and $L_j = (2^j - 1)(L - 1) + 1$ is the length of the scale τ_j wavelet filter.

Although the wavelet covariance decompose the covariance between two stochastic processes on a scale-by-scale, in some situations it may be beneficial to normalize the wavelet covariance by the variability inherent in the observed wavelet coefficients. The wavelet covariance at scale τ_j can be expressed as follows:

$$\gamma_{XY}(\tau_j) = \text{Cov}_{XY}(\tau_j) = \frac{1}{\hat{N}_j} \sum_{k=L_j-1}^{N-1} \hat{W}_{j,k}^X \hat{W}_{j,k}^Y \quad (11)$$

Given that covariance does not take into account the variation of univariate time series, it is natural to introduce the concept of wavelet correlation.

The wavelet correlation is simply made up of the wavelet covariance for $\{X_t, Y_t\}$ and wavelet variances for $\{X_t\}$ and $\{Y_t\}$. The MODWT estimator of wavelet correlation can be expressed as follows:

$$\hat{\rho}_{XY}(\tau_j) = \frac{\text{Cov}_{XY}(\tau_j)}{\hat{\sigma}_X^2(\tau_j) \hat{\sigma}_Y^2(\tau_j)} \quad (12)$$

As with the usual correlation coefficient between two random variables, $|\hat{\rho}_{XY}(\tau_j)| \leq 1$. The wavelet correlation is analogous to its Fourier equivalent, the complex coherency (Gençay et al (2002, p: 258)).

The wavelet cross-correlation decomposes the cross-correlation between two time series on a scale-by-scale basis thereby making it possible to see how the association between two time series changes as a function of time horizon. Gençay et al (2002) define the wavelet cross-correlation as:

$$\rho_{X,k}(\tau_j) = \frac{\gamma_{X,k}(\tau_j)}{\sigma_1(\tau_j) \sigma_2(\tau_j)} \quad (13)$$

where $\sigma_1^2(\tau_j), \sigma_2^2(\tau_j)$ are, respectively, the wavelet variances for $x_{1,t}$ and $x_{2,t}$ associated with scale τ_j and $\gamma_{X,k}(\tau_j)$ the wavelet covariance between $x_{1,t}$ and $x_{2,t-k}$ associated with scale τ_j . Just as the usual cross-correlation is used to determine lead-lag relationships between two processes, the wavelet

cross-correlation should be able to provide a lead-lag relationship on a scale by scale basis (Gençay et al (2002)).

3. Data description and basic statistics

The analysis was conducted using monthly data for the interest rate of American Treasury securities at 3-month constant maturity provided by the Federal Reserve and the exchange rate between USD and EURO. The closing S&P500 index is used as the indication of the stock price fluctuation. Empirical analysis covers the period from January 1990 to December 2008 providing 228 observations in total. In this study, we investigate the returns series $r_t = \ln(p_t) - \ln(p_{t-1})$. Initially Table I shows some brief summary statistics for the returns series of interest rate, exchange rate and stock index respectively. From the Table I, we make the following observations. (a) The mean of returns series is equal to zero for all series. (b) Interest rate returns have higher standard deviation than exchange rate and stock index returns showing that the interest rate has higher volatility than the exchange rate and than the stock market. (c) Monthly returns of interest rate tend to have high excess kurtosis.

Both series appear to have similar characteristics, in terms of mean and variation, but a more thorough description is available to use through a multi scale analysis.

Table 1. Descriptive Statistics for returns series

	min	max	mean	std.dev	skewness	kurtosis
Interest rate	-1.85	0.3	-0.02	0.17	-7.37	68
Stock index	-0.06	0.08	0	0.02	0.14	0.16
Exchange rate	-0.18	0.11	0	0.04	-0.88	2.04

4. Multi Scale Analysis

We apply the maximal overlap discrete wavelet transform to the monthly returns for the three series using the Daubechies (D) wavelet filter of length $L = 4$, that is $D(4)$, based on four non-zero coefficients Daubechies (1992), with periodic boundary conditions. The application of the translation invariant wavelet transform with a number of scales $J = 5$ produces five vectors of wavelet filter coefficients, that is w_5, w_4, w_3, w_2, w_1 , and one vector of scaling coefficients, v_5 . Since we use monthly data, the wavelet filter coefficients, $w_{5,k}, \dots, w_{1,k}$, represent progressively finer scale deviations from the smooth behavior, and correspond to 32 – 64, 16 – 32, 8 – 16, 4 – 8 and 2 – 4 months period, respectively.

Table 2. Multi scale Granger Causality

	S_5	D_1	D_2	D_3	D_4	D_5
FX $\rightarrow$ IR	3.935* (0.001)	1.850* (0.038)	1.2413 (0.2489)	1.6985 (0.0833)	1.1638 (0.2918)	1.6188 (0.0565)
SM $\rightarrow$ IR	2.910* (0.002)	1.405 (0.159)	1.2349 (0.2533)	1.6506 (0.0950)	1.9651* (0.0113)	2.8365* (0.0002)
IR $\rightarrow$ FX	1.225 (0.276)	1.700 (0.063)	2.0217* (0.0183)	2.7712* (0.0032)	3.0075* (0.0001)	1.9646* (0.0126)
SM $\rightarrow$ FX	2.087* (0.0271)	2.156* (0.013)	2.8408* (0.0007)	2.3289* (0.0130)	1.4176* (0.1202)	2.4540* (0.0012)
IR $\rightarrow$ SM	2.1934* (0.019)	1.399 (0.162)	1.1638 (0.3065)	1.1361 (0.3371)	2.1623* (0.0044)	2.4685* (0.0011)
FX $\rightarrow$ SM	1.653 (0.0934)	1.480 (0.127)	1.2736 (0.2273)	2.0022* (0.0349)	1.4643 (0.1003)	1.2351 (0.2348)

Note – S_5 is the original data transformed by the wavelet filter $D(4)$. The significances levels in parentheses.

* Significant at the 5% level.

The main purposes of this paper are to examine the lead-lag relationship and cross-correlation between interest rate, stock market and exchange rate over the various time scales using wavelet analysis. To examine the lead-lag relationship in wavelet analysis, first is tested for Granger causality up to level 5. The results of the Granger causality tests are reported in Table 2. As can be seen in Table

2, the stock market and interest rate show a feedback relationship only at higher scales (D_4 and D_5). The results show also, the unidirectional causality from stock market to exchange market in various time scales and from interest rate to exchange market at scales D_2 , D_3 , D_4 and D_5 .

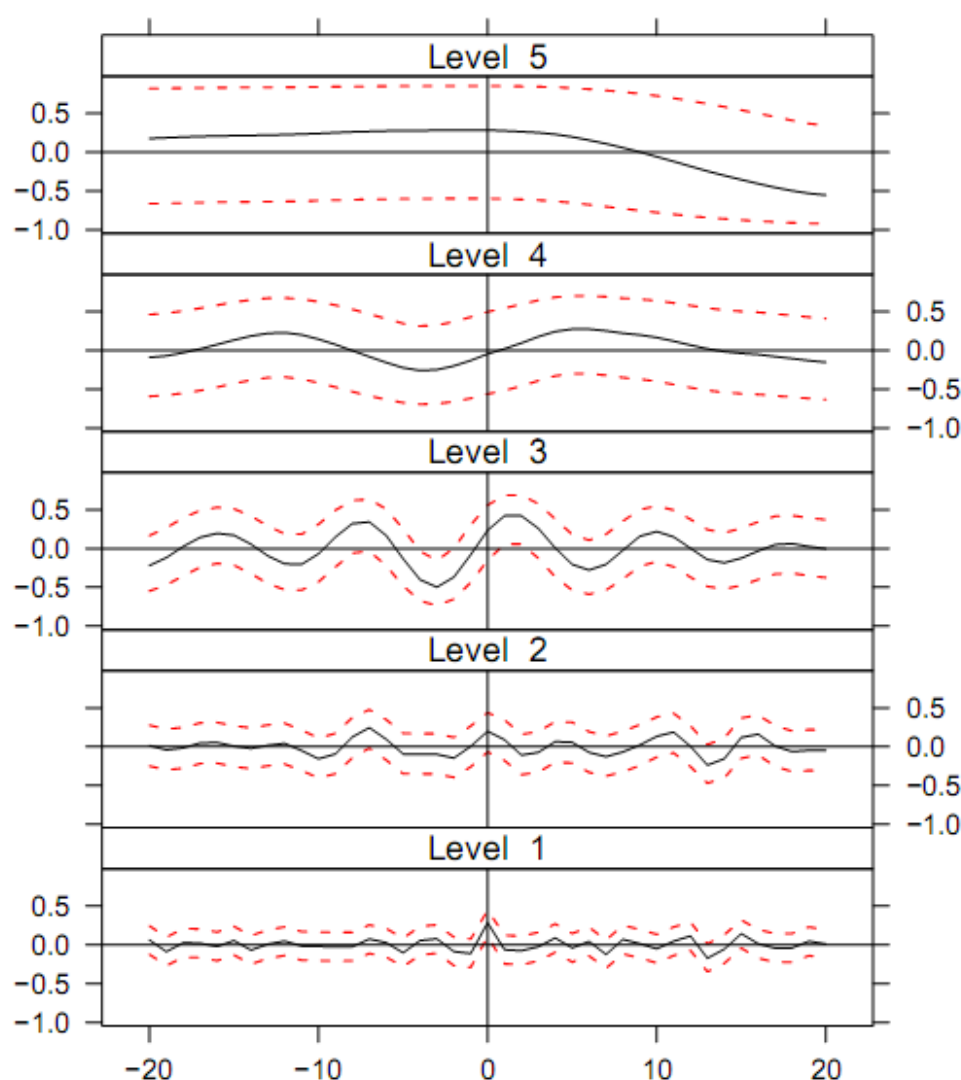


Figure 1. Wavelet cross-correlation between Interest rate and Exchange rate returns

For the second purpose of our paper (cross-correlation in various time scale), we examine the cross-correlation between the stock market, interest rate and exchange rate in various time scale. In Figures 1, 2 and 3 we report the estimated wavelet cross-correlations coefficients and the corresponding approximate confidence intervals against time leads and lags for all scales between the interest rate returns and exchange rate returns, interest rate returns and stock index returns and between exchange rate returns and stock index returns respectively. Figure 1 shows that, for all scales, the relationship between interest rate and exchange rate is generally not significantly different from zero at all leads and lags (slightly significant at scale 3). This means that interest rate returns and exchange rate returns in this period were independent and historical information of interest rate was not significantly predictive for exchange rate. In Figure 2, we report that at shortest scales, i.e. 1–3, the relationship between interest rate and stock index is not significantly different from zero. On the other hand, at the coarsest scales, i.e. 4–5, we report a significant positive relationship between the two series. Also, at scale 5, we underline a positive leading relationship between interest rate and stock returns, and we note the asymmetry in the wavelet cross-correlation sequence. At the fifth scale (associated with associations of 32 to 64 months), the interest rate is positively correlated with the stock return at a lead 10 months but not at a lag of 10 months. This figure also shows that most of

significant coefficients have positive values. This indicates that the interest rate appreciation (depreciation) was associated with a fall (rise) in stock index. The wavelet cross-correlation provides that exchange rate returns and stock index returns are independent at short horizons (high frequencies).

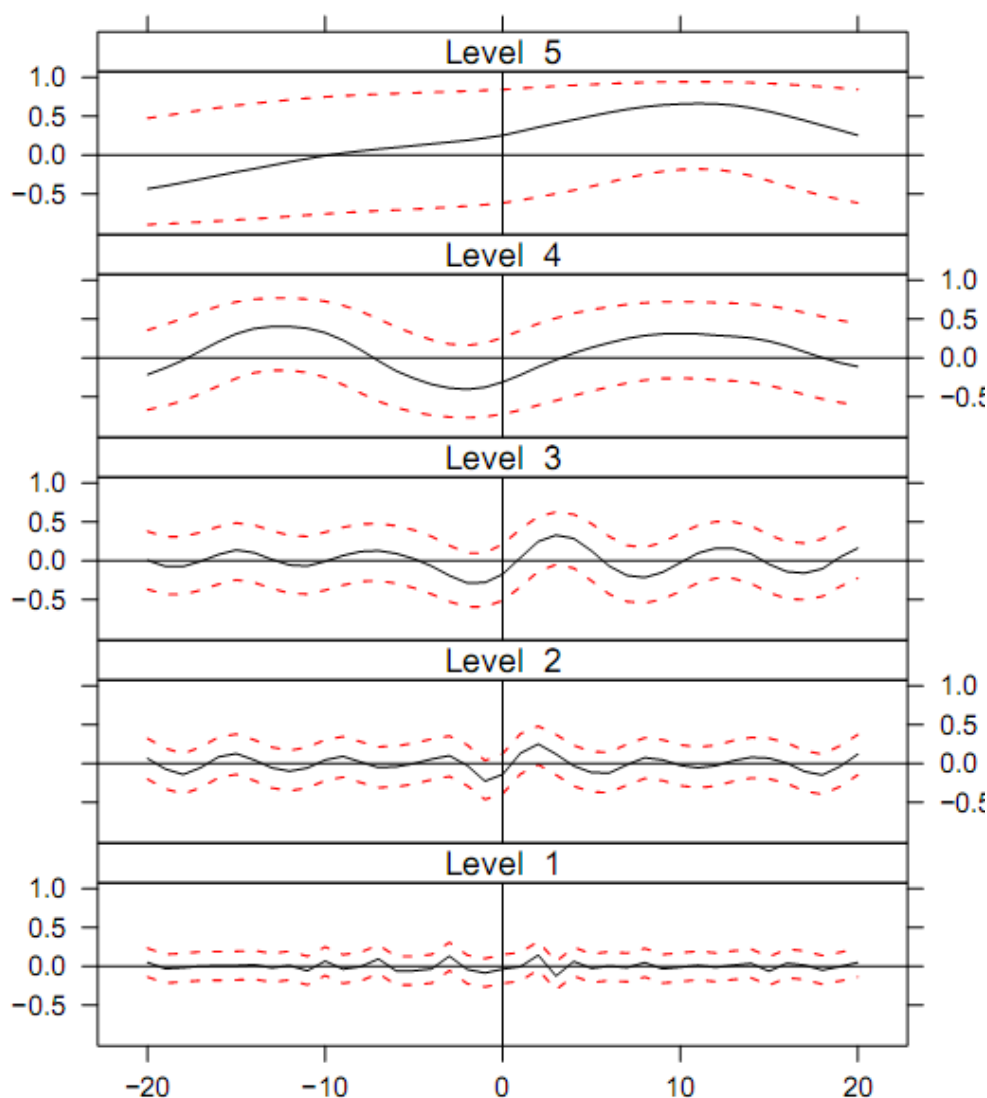


Figure 2. Wavelet cross-correlation between Interest rate and Stock index returns

The Figure 3 also shows a significant relationship between the two series only at coarsest scale. We note also that significant coefficients have positive values at leads and negative values at lags (scale 5). This means the existence for a bidirectional relationship between the two series at long horizons.

5. Conclusion

In this paper we apply a wavelet multi-scaling approach based on a maximum overlap discrete wavelet transform to investigate the relationship between interest rate, exchange rate and stock price over different time scale. Through a scale by scale decomposition of the cross-correlation between two time series, we try to shed some light on the scaling properties for the relationship at different time horizons. The main results are summarized as follows:

- The wavelet cross-correlation analysis shows that the relationship between interest rate and exchange rate is not significantly different from zero at all leads and lags and at all scales.
- The relationship between interest rate and stock index is significantly different from zero only at the coarsest scales, i.e. 4–5, which correspond to longer horizons. The analysis provides evidence about the finding that interest rate returns are leading stock index returns.

- Only at low frequencies (longer horizons), we remark a significant relationship between exchange rate and stock index at this period.

In general it seems that the interest rate and exchange rate series are generally quite independent at the period of studies and at all scales. There was, however a possible unidirectional causality running from interest rate to the stock price but not vice versa at highest scales. Therefore, our results show that a possible bidirectional causality running between exchange rate and stock index only at longer horizons.

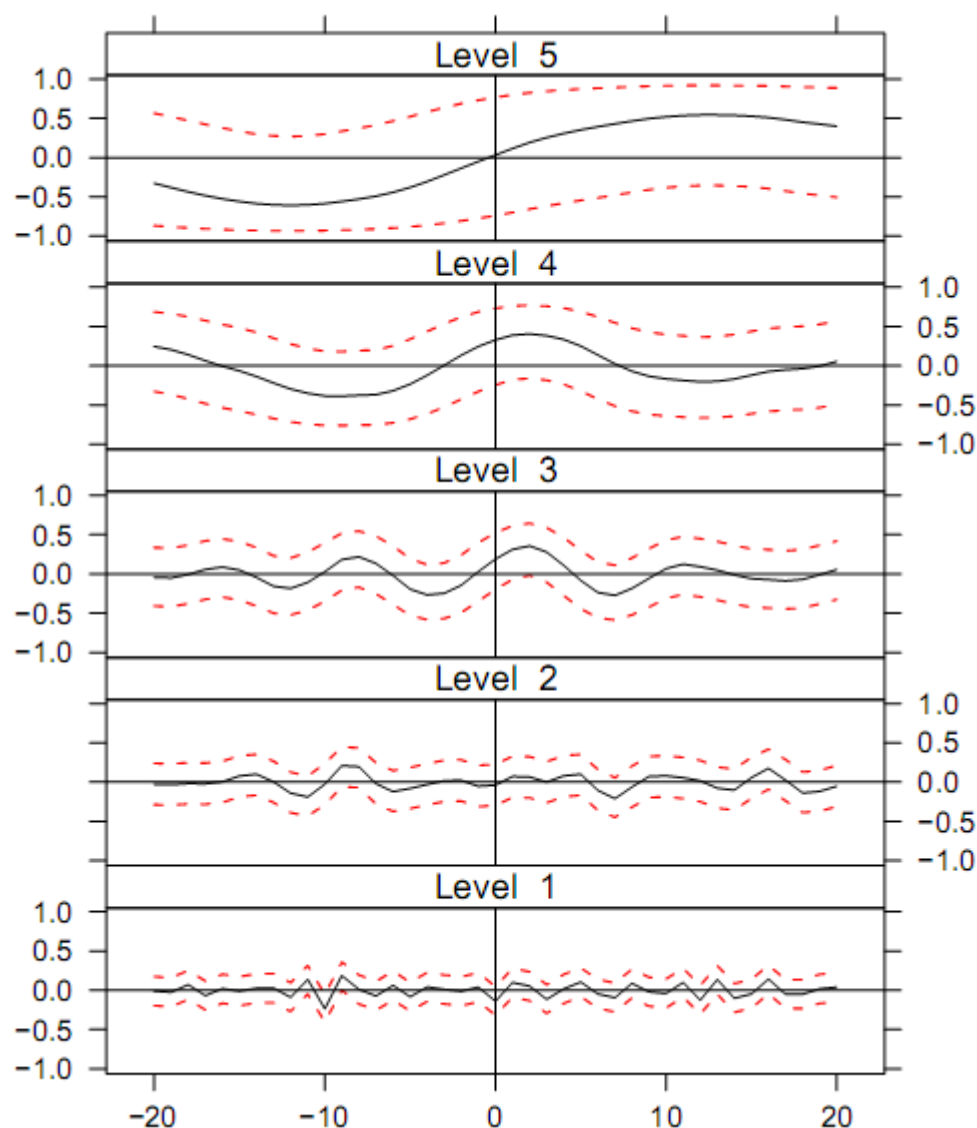


Figure 3. Wavelet cross-correlation between Exchange rate and Stock index returns

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