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Review of Different Methods for Siting and Sizing Distributed Generator

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ABSTRACT

Distributed generator has been confirmed to be a means by which installation of a new centralized generator can be delayed or postponed in the era of deregulation. It helps to minimize line losses and improve voltage profile. The benefits associated with distributed generator can be greatly harnessed when DG is optimally sited and sized within power system network. In this study, different methods used by previous researchers were extensively reviewed. The summary of the methodologies from previous works were itemized one after the other. This was done under the impact of distributed generator in a network. These methodologies are summaries under three categories namely; analytical, meta-heuristic and hybrid optimization methods. All the methods are based on solving non-linear load flow solution except the Inherent Structural Network Topology. In this review work, inherent structural network topology is recommended for further studies to solve power system problems.

Keywords: Distributed Generator, Site, Size, Line Losses, Reliability

JEL Classifications: C63, Q48

1. INTRODUCTION

A distributed generator (DG) is often referred to as "mini generation station" or "dispersed generator" which is often directly connected to the distribution network. It can either be a renewable or a non-renewable energy source (Jain et al., 2011). Distributed generator (DG) is a possible substitute for installing a new centralized generation station, particularly in the era of deregulation and it has several benefits such as low investment risk and short installation period (Gissey et al., 2021; Khaligh and Buygi, 2020; Martinez-Bolanos et al., 2020; Moret et al., 2020; Yang et al., 2020), modules with small capacity that can monitor variations in load very closely (Jiang et al., 2020; Mahmud et al., 2020; Somefun et al., 2020), small physical size suitable for installation at proximity to users' end, and availability of a large range of DG technologies (Nagaballi and Kale, 2020; Yang et al., 2021). However, it is important to note that when DGs are to be introduced into a distribution network, consideration should be to optimally site and size DG units otherwise, there will be an

increase in power losses and a decrease in reliability levels (Balu and Mukherjee, 2020; Deb et al., 2020; Farzinfar et al., 2020; Galgali et al., 2021; Kizito et al., 2020; Manna and Goswami, 2020; Shuaibu et al., 2020; Truong et al., 2020). In recent time, there have been rise in the study of distributed generation in power system analysis. The main reasons for the significant increase of interest in the direction of DG are summed up as follows (Ackermann et al., 2000; Ackermann et al., 2001; Carpinelli et al., 2001; Shayeghi and Alilou, 2021):

- i. Little or no cost is attributed to transmission and distribution because DG units are installed closer to the load point
- ii. Availability of plants ranging from few KW to tens of MW of different DGs
- iii. The introduction of DG will continue to increase the avoidable cost due to the introduction of additional transmission and distribution gadgets
- iv. Small generators can easily be located
- v. Natural gas, which is usually used as fuel in DG(s), is ubiquitously distributed

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- vi. It has low investment risk and requires short installation times
- vii. Increase in high reliable electricity to the consumers
- viii. DG plants yield fairly good efficiency
- ix. Constraints on the introduction of new transmission lines and distribution lines
- x. Electricity market liberalization
- DG provides a flexible way to select a wide range of combinations of cost and reliability.

In this present work, an extensive review of different DG technologies, impact of DG, and previous methodologies employed in siting and sizing DG is carried out.

1.1. Distributed Generation (DG) Technologies

The technologies implemented in DG consist of wind energy, solar energy, biomass, small gas turbines, small-hydro-power, fuel cells, micro-turbines, etc. (Ackermann et al., 2000; Ackermann et al., 2001; Kishinevsky and Zelingher, 2003; Lasseter, 1998; Rahman, 2003). Integration of DGs into an existing network can affect system operation in some ways such as losses, stability, voltage profile, and reliability (Agajie et al., 2021; Hafez et al., 2021; Yaghoubi-Nia et al., 2021). Moreover, environmental benefits given by the integration of renewable energy sources (RES)-based DGs are regarded as the main motivating force for the rise preference in the direction of these technologies (Paliwal et al., 2014). Usually, diesel generators are also considered as distributed generators owing to the fact that they are highly reliable with a low capital cost. For some years now, DG technologies have experienced substantial advancement. It is generally categorized as a non-renewable-based energy source, renewable-based energy source, and those which are referred to as electrical-based energy storage and load control strategies classification. The latter is not generating sources per se but they expedite efficient utilization of generated power. There are various DG technologies (Saad Al-Sumaiti et al., 2020; Willenberg et al., 2020; Zhou et al., 2020) and they are as depicted in Figure 1.

Table 1 shows the list of some DG technologies' types. For clarification, they are grouped into two major sections: (a) Non-renewable energy technologies and (b) renewable energy-based

technologies. These are evaluated based on numerous constraints i.e., load size, the amount of fuel consumed, the efficiency of the electrical side, efficiency of the whole system, cost of installation, the maintenance cost of the whole system, peak saving, reliability, power quality, and green power.

1.2. Applications of DG

User's requirements determine specific applications of a DG. The most common technological applications of DG are as follows (Ackermann et al., 2001; Arfeen et al., 2019; Dehghani et al., 2020; Ghanbari et al., 2018; Huy et al., 2020; Ismael et al., 2019; Kim et al., 2019; Saim et al., 2021; Wang et al., 2020):

- Base load i.e., to supply emergency or standby power to the network
- ii. Peak load i.e., to augment the available supply from utility to meet total load demand during peak period
- iii. Back-up supply and support to the distribution network i.e., to make supply available in case of fault.

1.3. Benefits of DG

Benefits of DG are classified under three categories namely; technical benefits, economical benefits, and environmental benefits (Khoa et al., 2006; Nagaballi and Kale, 2020; Nezhadpashaki et al., 2020).

1.3.1. Technical benefits

Technical benefits include the following:

- i. It reduces losses on the line
- ii. It aids better voltage profile
- iii. Increase in overall energy efficiency
- iv. Reliability and system security enhancement
- v. It improves the quality of power
- vi. Provides relief for transmission and distribution against congestion.

1.3.2. Economic benefits

Economic benefits include the following:

- i. It postpones facilities investments and system upgrades
- ii. It reduces operation and maintenance costs of PV and wind type of DG technologies

Figure 1: Distributed generation technologies

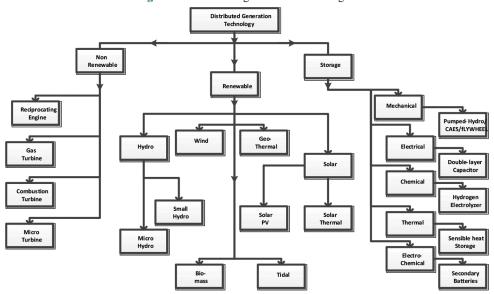


Table 1: Distributed generator Technologies (Viral and Khatod, 2012)

Serial	Type of	Fuel used	Size (KW)	EE (%)	OE (%)	IC	TMC	PS/Rty/	Green
number	technologies					(\$/KW)	(\$/KW)	PwQty	power
a. Nonrenewable energy-based technologies									
1	Engines with piston	Natural gas/ diesel	From 3 to 6000+	From 30 to 43	From 80 to 85	From 600 to 1200	From 0.005 to 0.015	Certain	Not at all
2	Combustion gas turbine	Gas/diesel	From 0.5 to 30,000+	From 21 to 40	From 80 to 90	From 400 to 900	From 0.004 to 0.010	Certain	Not at all
3	Small turbine	Natural gas/ bio-gas/ propane	From 30 to 1000	From 14 to 30	From 80 to 85	From 1200 to 1700	From 0.0018 to 0.015	Certain	Not at all
4.	Mix fuel cell	Ethenol/ natural gas/	From 400 to 20,000	From 35 to 55	From 80 to 85	From 4000 to 5000	From 0.0019 to 0.0153	Certain	Not at all
5	Automotive fuel cells	phosphoric acid/propane	From 30 to 60	From 30 to 55	From 80 to 90			Certain	Not at all
6	Micro fuel cell	Ethenol/ natural gas/ propane	From 1 to 300	From 30 to 50				Certain	Not at all
7	Small CHP	Heat space/ water	From 1 to 10	30 upward	From 75 to 89	From 500 to 845	-	Certain	Not at all
b. Renewa	able energy-based	technologies		-					
1	Wind	Wind	From 0.2 to 3000	NA	From 50 to 80	-	-	Not at all	Certain
2	Photovoltaic systems	Sun	From 0.02 to 1000+	NA	From 40 to 45	From 4500 to 6000	-	Not at all	Certain
3	Biomass gasification	Biomass	From 100 to 20,000		From 15 to 25	From 60 to 75	From 1500 to 3000	Not at all	Certain
4	Small Hydro-power	Water	From 5 to 100,000	NA	From 90 to 98	From 10,000 to 15,000	-	Not at all (PS)	Certain
5	Geothermal	Hot water	From 5000 to 100,000	From 10 to 32	From 35 to 50	-	-	Not at all	Certain
6	Ocean energy	Ocean wave	From 100 to 1,000	-	-	-	-	Not at all	Certain
7	Solar, thermal	Sun and Water	From 1000 to 30,000	From 30 to 40	From 50 to 75	-	-	Not at all	Certain
8	Battery storage	-	From 500 to 5000	NA	From 70 to 75	From 100 to 200	-	Certain	Certain

EE: Electrical efficiency, OE: Overall efficiency, IC: Installed cost, TMC: Total maintenance cost, PS: Peak saving, Rty: Reliability, PwQty: Power quality, NA: Not applicable

- iii. It boosts productivity
- iv. It reduces cost owing to healthcare by avoiding environmental pollution
- v. It reduces the cost of fuel owing to an increase in overall efficiency
- vi. It reduces the required reserve and its cost implications
- vii. It reduces the cost of operation owing to peak saving
- viii. It aids security for the crucial loads.

1.3.3. Environmental benefits

Environmental benefits include the following:

i. It reduces pollutants due to emission

It encourages a RES-based generation unit.

2. METHODOLOGY

This study employs review approach by summarizing different methods used in siting and sizing distributed generators in a power system. Distributed generation has several positive impacts on power system network such as improved voltage profile, increased system reliability level, minimized Total Harmonic Distortion (THD) (Ganivada and Venkaiah, 2014), minimized cost of electricity (Gupta et al., 2015), lower short-circuit

level (Asgharian and Genc, 2016), relieving transmission and distribution congestion, minimized line losses, etc. (Shomefun et al., 2018). However, with so many positive contributions from distributed generation, it is important to note that distributed generation must be optimally sited and sized to achieve the intended results (Jain et al., 2010; Jain et al., 2011; Musa et al., 2014).

2.1. Review of Loss Minimization and Voltage Profile Improvement

Technical losses are present in every stage of electrical power systems i.e. generation, transmission, and distribution, and they are caused by harmonics distortion, unbalanced loading, loss due to ageing and poor standard of equipment, etc. (Abdulkareem, 2016). In order to reduce losses between transmission and distribution centres, the power generated in the generation stations is maintained at high voltage before being transmitted to minimize losses on the transmission line. But this does not reduce losses between distribution stations and consumers i.e. the end-users. It is also important to note that voltage instability is one of the foremost problems in electrical power systems. Voltage profile stability is essential because variation in load demand leads to variation in voltage level. A decrease in voltage level leads to

an increase in reactive power demand, but if the reactive power demanded cannot be supplied, the voltage level will continue to drop until total blackout results. However, from previous research works, it was recorded that installation of distributed generators close to load centres can reduce losses between distribution centres and consumers' points with voltage profile improvement (Gupta et al., 2015). With such achievement by DG, several researchers have demonstrated different techniques to size and site single and multiple DGs on various distribution networks. Some of which are discussed as follows:

2.1.1. Analytical methods

Analytical methods are techniques that enable researchers to examine complex relationships between different variables used in modelling systems.

2.1.1.1. Exact loss formular

According to (Nweke et al., 2016), the exact loss formula is the most effective method when a single DG unit is to be placed to supply only active power. The exact loss formula is expressed as;

$$P_{Loss} = \sum_{\substack{k=1\\i-1}}^{n} \left[\alpha_{ki} (P_k Q_i + Q_k P_i) + \beta_{ki} (Q_k P_i - P_k Q_i) \right]$$
 (1)

where

$$\alpha_{ki} = \frac{\mathbf{r}_{ki}}{|V_k||V_i|} \cos(\delta_k - \delta_i)$$

$$\beta_{ki} = \frac{\mathbf{r}_{ki}}{|V_k||V_i|} \sin(\delta_k - \delta_i)$$

 $V_k < \delta_k$ = voltage magnitude and angle at the *k-th* bus;

 r_{ki} =line resistance between bus k and bus i;

 V_{i} and V_{i} = bus k and bus i voltages;

 P_k and $P_i = k - th$ and i - th active power injections buses;

 Q_k and $Q_i = k - th$ bus and *i-th* reactive powers injection buses; and

n=total number of buses.

In order to achieve maximum power injected at bus k, equation (1) is differentiated with respect to P_k and equated to zero to give

$$\frac{\partial P_{Loss}}{\partial P_{L}} = 2\sum_{i=1}^{n} (\alpha_{k} P_{i} - \beta_{k} Q_{i}) = 0$$
 (2)

This implies that

$$\alpha_{ki}P_i - \beta_{ki}Q_i + \sum_{k=1}^n (\alpha_k P_i - \beta_k Q_i) = 0$$
(3)

$$P_{k} = \frac{1}{\alpha_{ki}} \left[\beta_{ki} Q_{i} + \sum_{\substack{i=1\\i \neq k}}^{n} (\alpha_{k} P_{i} - \beta_{k} Q_{i}) = 0 \right]$$

$$\tag{4}$$

where P_k is the maximum active power needed to be injected at node k for minimal loss. It is the variation in active power generation and active power demand at the kth node. That is

$$P_{\nu} = (P_{DG} - P_{D}) \tag{5}$$

where P_{DG} is the injected active power from the installed DG at the k-th node and P_D is the required load at the kth node. Equating equations (4) and (5) results in equation (6). Equation (6) gives the definite optimal size of the DG for loss minimisation on the network.

$$P_{DGk} = P_D + \frac{1}{\alpha_{ki}} \left[\beta_{ki} Q_i + \sum_{\substack{i=1\\i\neq k}}^n (\alpha_k P_i - \beta_k Q_i) \right]$$
 (6)

Results from these equations are subject to the fulfilment of the following three restrictions:

1. Load flow equations:

$$P_{Gk} - P_{Dk} - V_k \sum_{i=1}^{ln} \left[G_{ki} \cos(\delta_k - \delta_i) + B_{ki} \sin(\delta_k - \delta_i) \right] = 0$$

$$Q_{Gk} - Q_{Dk} - V_k \sum_{j=1}^{ln} \begin{bmatrix} G_{ki} \sin(\delta_k - \delta_i) + \\ B_{ki} \cos(\delta_k - \delta_i) \end{bmatrix} = 0$$

2. Restriction on voltage: Restriction on voltage (in pu) at each bus must satisfy the following inequality:

$$V_{min} \leq V_{\leq} V_{max} \tag{9}$$

This generally lies between 0.95 and 1.05.

3. DG Capacity: DG unit must be kept within the satisfactory threshold. The threshold is given as:

$$P_{DGimin} \leq P_{DGi} \leq P_{DGimax}. \tag{10}$$

$$Q_{DGimin} \leq Q_{DGi} \leq Q_{DGimax}. \tag{11}$$

where P_{DGimin} and P_{DGimax} are respectively the minima and maximum active power generations for DG size(measure in kW) whereas Q_{DGimin} and Q_{DGimax} are respectively the minima and maximum reactive power generations for DG size (measure in kVar).

The exact loss formula has been deployed by several researchers to achieve various results such as minimum active power loss with improved voltage profile (Nweke et al., 2016; Rani and Devi, 2012), minimum total active and reactive power losses, and improved voltage profile at the injection distribution system, loss reduction, however, different load models were not considered.

2.1.1.2. Loss sensitivity factor

Authors in (Gözel and Hocaoglu, 2009) used the loss sensitivity factor to determine the site and size of DG in order to minimise losses on the radial distribution networks. This method (loss sensitivity factor) uses bus-injection to branch-current (bibc) and branch-current to bus-voltage (bcbv) matrices approach which is

based on the total injected current. In (Gözel and Hocaoglu, 2009), the total power losses mathematical expression was very essential in defining the optimum DG size and total system losses. The constraint on voltage magnitude is given as 1 ± 0.05 pu.

The step-by-step summary of this approach is given as follows (Gözel and Hocaoglu, 2009; Teng, 2000):

Step 1. Base case load flow simulation.

Step 2. Determination of optimum DG size for each bus with exception of the reference bus using

$$P_{dgi} = P_i + P_{loadi} \tag{12}$$

where

P_i is the active power to be injected at the bus i, and is expressed as

$$P_{i} = \frac{|V_{i}|[Z]^{T}[dPBIBC_{i}](cos\theta_{i}[re\ dI_{i}] + sin\theta_{i}[im\ dI_{i}])}{[Z]^{T}\ dPBIBCi(:,i-1)}$$
(13)

Step 3. Compute total power losses using equation (14) for each bus with respect to optimum power to the bus.

$$P_{loss} = \sum_{i=1}^{nb} |B_i|^2 . Z_i = [Z]^T [BIBC].[I]^2$$
 (14)

Step 4. Select the bus with minimum power loss after DG has been added.

Step 5. Determine the bus voltages using equation (15) and ascertain that the obtained voltages are within specified constraints.

$$[\Delta V]_{(n-1)\times 1} = [BCBV] \times [BIBC] \times [I]$$
(15)

where *BCBV* matrix gives the relationship that exists between bus voltages and branch currents. The elements of the matrix *BCBV* contain the branch impedances. An algorithm for developing matrix *BCBV* can be found in Teng (2000).

Step 6. For any bus violating constraints, DG is omitted from that particular bus and return to Step 4.

2.1.1.3. Power stability index (PSI)

Authors in (Aman et al., 2012) developed a power stability index with the aim of achieving stable node voltages based on the most crucial bus in the network that can cause voltage instability considering the amount of current being drawn by the load. The authors worked on visualizing the impact of DG on system losses, voltage profile and voltage stability. The authors used two bus systems to develop the index as shown in Figures 2 and 3. From Figure 2, equations 16 through 18 are obtained

$$S_I = P_I + jQ_I = V_I I_r^* \tag{16}$$

Figure 2: A two-bus network

$$\begin{array}{c|c} & I & V, \angle \delta, \\ \hline V_1 \angle \delta_1 & |Z| \angle \theta = r + jx & \\ \hline & P_L + jQ_L \end{array}$$

$$\overline{V_s} = \overline{V_r} + \overline{I_r Z} \tag{17}$$

where

$$I_r = \frac{\left(P_L\right) - j\left(Q_L\right)}{V_*^*} \tag{18}$$

From Figure 3, the following equations are obtained

$$I_{r} = \frac{(P_{L} - P_{G}) - j(Q_{L} - Q_{G})}{V_{r}^{*}}$$
(19)

By substituting I_r from equation (19), into equation (17) and separating into real and imaginary parts will give:

$$P_{L} - P_{G} = \frac{|V_{r}||V_{s}|}{V_{r}^{*}} Cos(\theta - \delta_{s} - \delta_{r}) - \frac{|V_{r}|^{2}}{Z} Cos(\theta)$$
(20)

$$Q_{L} - Q_{G} = \frac{|V_{r}||V_{s}|}{V_{s}^{*}} Sin(\theta - \delta_{s} - \delta_{r}) - \frac{|V_{r}|^{2}}{Z} Sin(\theta)$$
 (21)

Rearranging equation (20) gives

$$|V_r|^2 + A|V_r| + B = 0 (22)$$

where

$$A = -\frac{|V_r|}{Cos(\theta)}Cos(\theta - \delta_s - \delta_r)$$

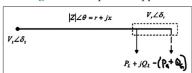
$$B = \frac{Z(P_L - P_G)}{Cos(\theta)}$$

Equation (22) is a quadratic equation, hence for stable node voltages, equation (22) be must real roots such that A^2 –4B>0. Hence, the Power Stability Index (PSI) is developed as shown in equation (23) as;

$$PSI = \frac{4r_{ij} \left(P_L - P_G \right)}{|V_i| \left(\theta - \delta_s - \delta_r \right)^2} \le 1$$
 (23)

Hence, for a stable operation condition, PSI must be less than 1. And a more stable condition is when PSI is very close to zero. Aman et al. (2012) used this index to find the optimum placement of DG by calculating the value of PSI for each line of the considered network, such that DG is placed at the bus having the highest value of PSI. However, the drawback in this approach

Figure 3: DG power support



is in the fact that the location of another distributed generator is dependent on the immediately previous one which results in the repetition of the procedure. The result of the PSI index was found to be in consonance with the Golden Section Search (GSS) algorithm.

2.1.1.4. Inherent structural characteristic indices

Authors in (Sikiru, 2014) developed inherent structural characteristic indices using atomic structure analogy with the aim of providing information on the structural relationship between buses for which load flow solution depends. This was further confirmed and extended by (Alayande, 2017). In their works, they started by considering Ohm's law

$$V=IZ$$
 (24)

Due to the fact that power system networks are structurally complex circuit, they represented the impedance *Z* with admittance *Y*. Therefore, equation 24 is written as

$$I=VY$$
 (25)

The *Y*-admittance matrix is then partitioned based on generators and loads as

$$Y = \begin{bmatrix} Y_{GG} & Y_{GL} \\ Y_{LG} & Y_{LL} \end{bmatrix}$$
 (26)

This partition was achieved by considering the three region of attraction between generator and load as a clue from electron-proton region attraction created by push-pull force between them. This then led to representation given in equation 27.

$$\begin{bmatrix} I_G \\ I_L \end{bmatrix} = \begin{bmatrix} Y_{GG} & Y_{GL} \\ Y_{LG} & Y_{LL} \end{bmatrix} \begin{bmatrix} V_G \\ V_L \end{bmatrix}$$
 (27)

By a little modification, the equation becomes

$$\begin{bmatrix} V_L \\ I_G \end{bmatrix} = \begin{bmatrix} Z_{LL} & F_{LG} \\ K_{GL} & A_{GG} \end{bmatrix} \begin{bmatrix} I_L \\ V_G \end{bmatrix}$$
 (28)

The matrix in equation 28 represents the entire attraction that exist between generators and loads. The strength of attraction between generator and load buses are observed through the eigenvalue decomposition.

$$A_{GG} = W \sum W^* = \sum_{i=1}^{r} w_i \sigma_i w_i^*$$
 (29)

Expanding equation 2.28 gives

$$V_G = A_{GG}^{-1} \Big[I_G - K_{GL} I_L \Big] \tag{30}$$

Substituting equation 30 in 29 yields

$$V_{G} = \sum_{j=1}^{r} \frac{w_{j} w_{j}^{*}}{\sigma_{j}} \left[I_{G} - K_{GL} I_{L} \right]$$
(31)

Hence, the bus associated with the smallest eigenvalue is the generator or load bus due to its reciprocal relationship with the load voltages.

2.1.2. Meta-heuristic methods

Meta-heuristic methods are high-level techniques designed to find or generate a partial search algorithm that possibly will provide a sufficient solution to optimisation problems especially with incomplete or imperfect information or limited computation capacity.

2.1.2.1. Backtracking search optimisation (BSA)

BSA is an innovative evolutionary optimisation program very close to Particle Swarm Optimisation and Genetic Algorithms. BSA is a dual-population-based optimisation program that uses two sets of populations in its global search approach namely; historical and current populations. BSA approach is unique in the sense that it minimises the objective function instead of maximising it as is the case in most other optimisation techniques. It consists of five stages explained as follows:

i. Initialisation

In this step, the program uses the dimension of the problem to initialise the population as shown in equation (32).

$$Pki\sim U(Min_{\iota}, Max_{\iota}) \tag{32}$$

ii. First selection

In this step, the previous population is used to obtain the direction of the search.

$$previousP_{ki} \sim U(Min_k, Max_k)$$
 (33)

The previous population is then shuffled in a random manner:

$$previousP:=permuting (PreviousP)$$
 (34)

iii. Mutation

In this step, the trial population is produced which is known as a mutant.

$$Mutant=P+F.(previousP-P)$$
 (35)

In equation (35), F regulates the size of the matrix in the direction of the search space.

iv. Crossover

In this step, the final population to be employed is produced. And is deployed to produce the target population.

v. Selection selection-II

In this step, the best fitness is selected to update the population which results in global minimisation.

Backtracking Search Optimisation has been deployed by (Gupta et al., 2015) to optimally locate highly important locations for DG placement and optimally size DGs by keeping active power loss to the barest minimum, with consideration to different load models. However, the reliability of the network was not considered.

2.1.2.2. Teaching and learning-based optimisation (TLBO)

Teaching and learning-based optimisation (TLBO) is a metaheuristic population-based program. It employs the ability of the teacher to teach and the student to learn. Compared to other metaheuristic algorithms like Genetic Algorithm (mutation and cross over probability), Particle Swarm Optimisation (C1, C2, Wmax, Wmin), etc., TLBO has no specific parameters in its approach. In the TLBO algorithm, random values are first generated to represent initial results which are referred to as learners. The level of understanding of each learner is assessed based on objective function (O-F). The learner, who is highly intelligent (meaning best of result), is preserved as a tutor and the rest of the solutions as learners (Rao and Patel, 2013). It consists of two phases namely; the tutor phase and the learner phase.

The tutor phase is when knowledge is passed from the tutor (best result) to students.

$$Diff_{ki}^{mean} = rand (X_{ti} - (T_t * M_i))$$
(36)

$$X_{ki}^{new} = X_{ki}^{old} + Diff_{ki}^{mean} \tag{37}$$

Replace X_{ki}^{old} with X_{ki}^{new} only if X_{ki}^{new} results in a more improved solution than X_{ki}^{old} .

During the learning stage, each learner tried to get a better understanding by learning from those with superior knowledge. For every learner, the partner is being generated randomly.

If
$$G(X_{i}) < G(X_{i})$$

$$X_{ki}^{new} = X_{ki}^{old} + random*(X_{ki}^{old} - X_{qi})$$

If
$$G(X_{ki}) > G(X_{ai})$$

$$X_{ki}^{new} = X_{ki}^{old} - random*(X_{ki}^{old} - X_{qi})$$

$$X_{ki}^{new}$$
 is selected if $G(X_{ki}^{new}) \leq G(X_{ki}^{old})$.

Authors in (Ganivada and Venkaiah, 2014) used TLBO to solve the problem of voltage profile, line loss, and line loading by obtaining optimal DG size and optimal location for DG placement. The author's model given in equation (38) minimises three indexes namely; loss, voltage, and MVA. However, in this work, DG technology used was not considered.

$$MinF = \begin{pmatrix} A*LOSS_{index} + \\ B*VOLTAGE_{index} + C*MVA_{index} \end{pmatrix}$$
(38)

where A,B,C are literally chosen values such that

$$A + B + C = 1$$
.

2.1.2.3. Cat swarm optimisation (CSO)

Cat swarm optimisation requires two processes to optimize its objective function. Like every other evolutionary optimisation technique, CSO also requires an initialisation process which, in this case, is for two parameters namely; position and velocity. The other process is the search process. In this process, two modes are considered namely; seeking mode and tracing mode.

Initialisation Process:

i. The position of each cat is first randomly initialized i.e. position of the ith cat in the D-dimensional space as

$$X_{i} = (X_{i1}, X_{i2}, \dots, X_{iD})$$
(39)

where *D* represents the dimension.

ii. The velocity of each cat is also initialized i.e. velocity of the ith cat in the D-dimensional space is given as

$$V = (V_{i1}, V_{i2}, ..., V_{iD})$$
 (40)

Search Process:

Seeking mode: Here, three features are important namely; sum total of dimension required for change, seeking boundary of the required dimension, and seeking from the stored data (Chu and Tsai, 2007).

In order to understand seeking mode, it is explained as follows: n copies of the present position of cat, are made, where

$$n = \text{store data} - 1$$
 (41)

one copy of the initial location is kept.

For each copy, randomly add or subtract the seeking boundary percentage from the initial values and replace the previous ones.

Calculate the best fit values of all contestants.

Tracing mode

In order to understand tracing mode, it is explained as follows:

The speed for each cat is updated (v_i, d) as shown below:

$$V_k^{j+1} = w \times V_k^j + r \times (GB^j - X_k^j)$$
(42)

where

w implies the inertia weight,

r is randomly distributed number between 0 and 1.

 GB_j is the optimum location of the whole population at *j-th* time of analysis.

 X_k^j is the location of the *k-th* cat in *j-th* analysis.

The speeds of the cats are restrained not to exceed the maximum range. In the event of a new speed outside the range, its value is set to be the same as the maximum limit.

The location of *k-th cat* is updated with the equation below;

$$X_k^{j+1} = X_k^j + V_k^{j+1} (43)$$

Authors in (Kumar et al., 2014) used the cat swarm optimization technique to optimally place multiple distributed generators in order to achieve reliability in distribution networks in terms of continuous power supply to healthy section. The authors further studied the influence of several distributed generators on bulk power to be transferred and loss minimization. However, optimal sizing and voltage limit were not considered.

2.1.3. Hybrid method

Hybrid method is simply the combination of two or more methods to achieve the same result that a singular method could have been used to achieve. However, the hybrid is usually employed because of its peculiarity to give a more reliable result without delay. Though the hybrid method is usually complex to deploy, it is worth going through the rigour when the accurate result is of necessity most especially for real-life situations.

2.1.3.1. Ranked evolutionary particle swarm optimisation (REPSO) algorithm

The Rank Evolutionary Particle Swarm Optimisation (REPSO) combines Evolutionary Programming (EP) with Particle Swarm Optimisation (PSO). EP is a seeking approach usually employed for both random selection and variation. It is based on a metaheuristic population EP uses the biological evolution of the natural processes to search for an optimal solution. EP makes use of several trials instead of a one-point to reveal possible answers to a problem. This is one of the advantages that help EP to escape being the confined local minimum. When the population of parents and offspring are combined, EP chooses survivals for the next generation by employing a tournament scheme. The survivours with the best fitness function in the population are thereafter held back as the latest parents of the succeeding generation, while the remaining are eliminated from the population. This action goes on up until the answer converges. This process of EP is deployed alongside particle swarm optimisation to attain a global optimum result under a short time frame. The advantages of hybridising these two methods in contrast to ordinary particle swarm optimisation are accuracy and speed (Musa and Adamu, 2013). By hybridizing evolutionary programming (EP) through the Ranking process to obtain a global result, the particle swarm optimisation is assisted to accomplish the best-guaranteed solution.

Authors in (Musa and Adamu, 2013) proposed REPSO to size distributed generation (DG). In their paper, a simple and effective approach for power loss reduction (PLR) value was employed to

first allocate DG before deploying REPSO. This was carried out by adding all power losses values at each branch using

$$P_{losses} = \sum_{j=1}^{n} \left| I_{j} \right|^{2} R_{j} \tag{44}$$

where

n =branches' total number:

 I_i = magnitude of current flowing; and

 R_{i} =line resistance.

The reduced power losses with respect to the introduction of the distributed generator are obtained by subtracting power losses in the network with DG installed from power losses without DG installation. These reduced power losses are expressed as shown in equation (46).

$$P_{losses-new} = \sum_{i=1}^{n} \left| I_i^{new} \right|^2 R_i \tag{45}$$

$$P_{losses-new} = \sum_{i=1}^{n} I_{j}^{2} R_{j} - 2k I_{j} I_{DG} R_{j} - k I_{DG}^{2} R_{j}$$
(46)

where

k=1 for network consisting of DG or else k=0.

Therefore, the power loss reduction result for bus j which contain DG is obtained by deducting equation (44) from equation (46);

$$PLR_{j} = P_{losses-new} - P_{losses}$$
 (47)

$$PLR_{j} = \sum_{j=1}^{n} (2kI_{j}I_{DG} + kI_{DG}^{2})R_{j}$$
(48)

Thus, the bus with the highest result for PLR is selected to be the best position for DG. DG current with the highest reduction of losses is obtained by differentiating equation (48) in connection with I_{DG} . Then equating the result obtained to zero. Hence, the current through the distributed generator is given as shown in equation (49):

$$I_{DG} = -\frac{\sum_{j=1}^{n} I_{aj} R_{j}}{\sum_{j=1}^{n} R_{j}}$$
(49)

This process continues for all the buses until the best solution is achieved. This helps to determine the best position for the distributed generator that yields the least loss on the network. The power generated from the distributed generated is obtained using the expression in equation (50).

$$P_{DG_i} = I_{DG^*} |V_i| \tag{50}$$

where

 $|V_i|$ = magnitude of voltage at bus i; and

 P_{DGi} = DG capacity that gives best result.

The best position for the distributed generator at bus *i* results in the least loss on the network.

2.2. Review of Stability of Electric Power System

An object is said to be stable if it goes back to its original position after being subjected to stress. In the same vein, an electric power system that comprises several systems interconnected together is said to be stable if it is capable of remaining in its normal or synchronous operating condition after having been subjected to disturbance. It is important to note that the electric power system network is susceptible to various disturbances such as short circuit, sudden loss of load, etc. However, these must be quickly attended to as soon as possible to prevent blackout or total outage. It is difficult to maintain synchronism between various parts of the power system because of the increasing interconnection of generating stations and load centres. This makes it very essential to consider stability during the planning stage of the power system or when an upgrade is needed. The stability study of the electric power systems is assessed by the impact of interruption on the electromechanical dynamic behaviour of the electric machines. It is of three types namely: steady-state, dynamic and transient stability (Kothari and Nagrath, 2011).

Power system instability is an important power system problem that must be attended to during planning, maintenance, or upgrade of the network. This is because instability is one of the major causes of blackouts (Kundur et al., 2004). In general, an electric power system is said to be stable for a given initial operating condition, if it is capable of recovering its normal or synchronous state of equilibrium operating condition after having been subjected to disturbance, so that the system will remain practically intact with most of the system variables bounded (Kundur et al., 1994; Kundur et al., 2004). This simply means that power system stability depends on the type of disturbance and initial condition of operation. Basically, power systems are subjected to both small and large disturbances. The continuous steady change in load demand can be referred to as small disturbances, and the system must be able to cope with the changes to function satisfactorily. It must also be able to withstand severe disturbances such as short circuit or loss of a large generator. However, power system cannot be designed to accommodate all possible large disturbances because of economic consideration and very low probability of such scenario happening. Hence, design contingencies are made for

situation of high probability of large disturbances occurrence (Kundur et al., 2004). Essentially, there are three aspects of system stability namely: rotor angle stability, frequency stability and voltage stability (Bahramipanah et al., 2011). Large disturbance angle stability which is recognized with transient stability is a highly subjective issue (Bahramipanah et al., 2011).

2.3. General Review of Previous Works on Distributed Generators

It is quite clear that the distributed generator (DG) offers several advantages (Malik et al., 2020). However, as good as it sounds, it is important to note that the benefit from the integration of DG is not without technical problems. These problems include islanding, best location, required size, etc. It is in solving these problems that maximum benefit from DG integration is obtained. For a few years, these problems have received much attention from power systems researchers. They considered different methods for locating DG units in distribution networks. Techniques for DG placement differ and they are dependent on the objectives and solution approach. Several works carried out in the planning of DG show that researchers have much interest in this area. Current and past researches have recommended several optimal location techniques varying from analytical to hybrid optimization techniques that have effectively sited and sized DG units (Abbagana et al., 2011; Abdelaziz et al., 2012; Acharya et al., 2006; Asgharian and Genc, 2016; Baran and Wu, 1989; Chen et al., 2015; Ganivada and Venkaiah, 2014; Gözel and Hocaoglu, 2009; Gupta et al., 2015; Jain et al., 2011; Kumar and Samantaray, 2014; Musa et al., 2014; Nweke et al., 2016). An analytical approach, based on the exact loss formula for optimal size and location considering loss minimization as an objective, was presented by Nweke et al. (2016). Likewise, optimization methods such as Differential Evolution (DE), Discrete Particle Swarm Optimization (DPSO), and Teaching and Learning-Based Optimization (TLBO) were presented in (Abbagana et al., 2011; Ganivada and Venkaiah, 2014; Musa et al., 2014) respectively, but none of these considered different types of loads and reliability indices. Authors in ref. (Jain et al., 2011) proposed a method based on voltage stability index and PSO to determine the optimal size of the distributed generator by considering multi-objective criteria for different categories of loads to minimize power loss, improve voltage profile and reduce greenhouse gas emission. Authors in ref. (Jamian et al., 2012) compared Rank Evolutionary Particle Swarm Optimization (REPSO) with Evolutionary Particle Swarm Optimization (EPSO) and Traditional Particle Swarm Optimization (PSO) to regulate DG output for optimal sizing. In ref. (Chen et al., 2015), Fuzzy Adaptive Hybrid Particle Swarm Optimization (FAHPSO) was implemented to minimize the comprehensive cost which consists of both the cost of power loss and the operation cost of primary devices (i.e. load transformer changers (LTCs), and capacitor bank switches). Authors in ref. (Kumar and Samantaray, 2014) considered only the optimal placement of multiple DG units using Cat-Swarm-Optimization (CSO) and composite reliability index.

Backtracking Search Optimization Algorithm in ref. (Gupta et al., 2015) and Teaching and Learning-Based Optimization (TLBO), PSO & GA in (Ganivada and Venkaiah, 2014) were used to determine the Optimal Site and Size of a distributed generator such that the system losses are minimum and voltage profile is maintained within limits. Authors in ref. (Asgharian and Genc, 2016) used the Goal attainment method to obtain optimum sizes and locations of DGs, capacitor, and static VAR so that the active power losses and the deviations in bus voltages, from their nominal values, are minimized. An estimation of total power losses of distribution networks for any new DG condition, using second-order power flow sensitivities, was discussed in (Ayres et al., 2014).

Abdelaziz et al. (2012) and Rafi and Dhal (2020) tried to reconfigure the distribution network in order to minimize losses on the line. Abdelaziz et al. (2012), network reconfiguration problem was solved for minimal loss using Ant Colony Optimization (ACO) algorithm implemented in the hypercube (HC) framework and random search musician-behaviour-inspired evolutionary algorithm, harmony search (HS). Baran and Wu (1989) on the other hand, used the simplified DistFlow method and Backward & Forward update of DistFlow to realize network reconfiguration in distribution systems by changing the status of sectionalizing switches for loss reduction or for load balancing in the system. Optimum power flow (OPF) and heuristic technique were used in (Gomes et al., 2006) to achieve optimum power flow for distribution system reconfiguration. Other researchers also look at the restoration of a distribution network with DG. Authors in (Chen et al., 2011) used a particle swarm algorithm based on twodimensional depth-coded for service restoration in a distribution network with DG in the event of a large-scale blackout. In ref. (Shahrin et al., 2012), Improved Genetic Algorithm (IGA) was used to integrate DG into the distribution network to compensate for the non-restored load.

Baran and Wu (1989) also tried to minimize losses in the distribution network by introducing a capacitor. The authors basically worked on the optimal size of the capacitor that can minimize real power loss in a radial distribution system of a giving load profile. The solution approach was based on the Phase I - Phase II feasible directions algorithm, with the assumption that the capacitors are already placed and that the distribution system has only one load profile.

3. RESULTS

In order to expand on the previous work done and find a simplified and straight forward approach to the problem of optimal siting and sizing of the distributed generator(s), another approach to identifying optimal site of DG should be investigated and explored. Example of such approach is Inherent Structural Network Topology (ISNT) which is based on the network interconnections between sources and loads (Ugranlı and Karatepe, 2012). This approach does not require the power flow equation but strictly adheres to basic mathematical rules and circuit principles on which the power flow equation depends.

The idea of the ISNT is based on the interconnection of buses which is described by the Z-impedance matrix (electrical connection). With the help of eigenvalue decomposition on the Z-impedance matrix, the sensitivity of each bus is determined. This proposed technique will aid power engineers in the proper planning and design of the power system as it does not depend on the load demand on the network. In the open literature, this technique is known as the Inherent Structure Theory of Networks (ISTNs). This theory can be applied to power quality studies (Carpinelli et al., 2001) and weak bus identification (Sikiru, 2014). However, this technique has not been fully explored; its application by most researchers has been limited to the Y-admittance matrix due to its sparse advantage. If Z-matrix of this formulation can be employed, network stability analysis can easily be introduced due the DG placement. Other major prevailing issues can be found in ref. (Shomefun et al., 2018). As an illustrated example from Aman et al. (2012), a 12-bus radial distribution network is considered and using the Inherent Structural Network Topology (ISNT) approach, the following result are obtained.

From the radial 12-bus feeder in ref. (Aman et al., 2012), bus 1 is the main substation bus which is also taking as the slack bus. The remaining eleven buses are load buses from which the optimal location of DG is obtained. Because the load buses are eleven in number, the result of the D_{LL} matrix is 11×11 as represented in Table 2. These buses are buses 2 through to 12. The result of the 11×11 D_{LL} matrix is subjected to singular value decomposition (SVD) to determine the eigenvalue for each of the load buses which is represented in Table 3. The eigenvalue is

Table 2: 11 \times 11 D₁₁ matrix of load bus with Schur complement of Z₁₁

Table 2. 11 ^ 11 D _{LL} matrix of load bus with Schur complement of Z _{LL}											
D_{LL} =	2	3	4	5	6	7	8	9	10	11	12
2	1.118	1.118	1.118	1.118	1.118	1.118	1.118	1.118	1.118	1.118	1.118
3	1.118	2.4622	2.4622	2.4622	2.4622	2.4622	2.4622	2.4622	2.4622	2.4622	2.4622
4	1.118	2.4622	4.6971	4.5069	4.7434	4.9244	4.6971	4.9244	4.5069	4.5069	4.6971
5	1.118	2.4622	4.6971	8.0777	8.3104	8.3104	8.2765	8.2765	8.0777	8.0777	8.0777
6	1.118	2.4622	4.6971	8.0777	9.424	9.424	9.3941	9.6209	9.1924	9.1924	9.1924
7	1.118	2.4622	4.6971	8.0777	9.424	10.5386	10.5119	10.5386	10.3078	10.3078	10.3078
8	1.118	2.4622	4.6971	8.0777	9.424	10.5386	15.0416	15.0416	14.8661	14.8661	15.0416
9	1.118	2.4622	4.6971	8.0777	9.424	10.5386	15.0416	20.9538	20.5548	20.7183	20.7183
10	1.118	2.4622	4.6971	8.0777	9.424	10.5386	15.0416	20.9538	23.7171	23.7171	23.7171
11	1.118	2.4622	4.6971	8.0777	9.424	10.5386	15.0416	20.9538	23.7171	25.2982	25.2982
12	1.118	2.4622	4.6971	8.0777	9.424	10.5386	15.0416	20.9538	23.7171	25.2982	26.6423

Table 3: Optimal location for distributed generator in Radial 12-bus feeder

Bus number	Eigen value	Rank position
2	118.5659	11
3	16.6256	10
4	3.9901	9
5	2.7467	8
6	1.9054	7
7	1.0492	6
8	0.908	4
9	0.908	5
10	0.3541	1
11	0.4474	2
12	0.4749	3

Table 4: Radial 12-bus feeder base case load flow result without distributed generator

Bus	Voltage	Angle	Load		Subst	tation
number	magnitude	Degree	kW	kVAr	kW	kVAr
1	1.0000	0	0	0	455.6689	413.0314
2	0.9944	0.0022	60	60	0	0
3	0.9892	0.0042	40	30	0	0
4	0.9808	0.0076	55	55	0	0
5	0.9703	0.0120	30	30	0	0
6	0.9670	0.0133	20	15	0	0
7	0.9643	0.0145	55	55	0	0
8	0.9561	0.0193	45	45	0	0
9	0.9482	0.0237	40	40	0	0
10	0.9455	0.0252	35	30	0	0
11	0.9446	0.0257	40	30	0	0
12	0.9444	0.0258	15	15	0	0
Total			435	405	455.6689	413.0314

thereafter ranked from the lowest to the highest and the load bus corresponding to the first ranked eigenvalue marks the optimal location for distributed generator installation. For this radial 12-bus feeder, the result from Table 3 shows that bus 10 is the optimal location for the distributed generator to be installed. However, according to the result from (Aman et al., 2012), bus 9 is taken as the location for DG. In order to justify the result obtained from the ISNT approach proposed in this study, the load flow result is obtained as shown in the following tables and figures.

Table 4 shows the load flow result through forward-backward sweep method. It is obvious that from the result that the load buses number 9, 10, 11, and 12 have a voltage profile outside the acceptable range of $1\pm5\%$. Also, considering the sequence of bus numbering (i.e. bus 1, bus 2,... bus 12) the first bus voltage outside the defined range is bus 9 which justify the result obtained by (Aman et al., 2012). However, in this study, utmost attention is given to optimal location for DG which will simultaneously result in improved voltage profile and minimal losses. This is justified by injecting 30 kW at buses 9 and 10 one after the other. From the base case forward-backward sweep result, the total active and reactive losses on the feeder are 17.448 kW and 6.8243 kVar respectively as shown in Table 5. By injecting 30 kW at buses 9, the voltage profile improves slightly and shown in Table 6 and the total loss by 16.60% for active loss and 15.86% for reactive loss. This can

Table 5: Radial 12-bus feeder total loss without distributed generator

uisti ibt	ittu gt		1.11	т.	
Line			ous and line	Line	eloss
		fl	ow		
From	To	kW	kVAr	\mathbf{kW}	kVAr
1		0	0		
	2	529.0168	220.222	2.9661	1.2347
2		60	60		
	1	-526.051	-218.9873	2.9661	1.2347
	3	453.3425	189.148	2.3878	0.9963
3		40	30		
	2	-450.955	-188.1517	2.3878	0.9963
	4	405.8607	169.9359	3.4096	1.4276
4		55	55		
	3	-402.451	-168.5083	3.4096	1.4276
	5	335.9244	140.0387	3.6275	1.5122
5		30	30		
	4	-332.297	-138.5265	3.6275	1.5122
	6	296.0798	123.2537	0.9869	0.4108
6		20	15		
	5	-295.093	-122.8429	0.9869	0.4108
	7	272.7383	113.5049	0.7728	0.3216
7		55	55		
	6	-271.966	-113.1833	0.7728	0.3216
	8	202.6121	55.9105	1.7289	0.4771
8		45	45		
	7	-200.883	-55.4334	1.7289	0.4771
	9	148.5436	42.0461	1.2158	0.3441
9		40	40		
	8	-147.328	-41.702	1.2158	0.3441
	10	100.9115	28.5625	0.2922	0.0827
10		35	30		
	9	-100.619	-28.4798	0.2922	0.0827
	11	61.2937	17.3274	0.0568	0.0161
11		40	30		
	10	-61.2369	-17.3114	0.0568	0.0161
	12	17.2879	4.9015	0.0037	0.001
12		15	15		
	11	-17.2842	-4.9005	0.0037	0.001
Total los	s			17.448	6.8243

Table 6: Radial 12-bus network load flow result with distributed generator at bus 9

uistributeu generator at bus >										
Bus	Voltage	Angle	Load		Subst	tation				
number	magnitude	degree	kW	kVAr	kW	kVAr				
1	1	0.0000	0	0	423.0621	412.069				
2	0.9947	0.0023	60	60	0	0				
3	0.9898	0.0045	40	30	0	0				
4	0.9821	0.0080	55	55	0	0				
5	0.9724	0.0127	30	30	0	0				
6	0.9694	0.0142	20	15	0	0				
7	0.9670	0.0155	55	55	0	0				
8	0.9600	0.0205	45	45	0	0				
9	0.9537	0.0253	10	40	0	0				
10	0.9510	0.0268	35	30	0	0				
11	0.9501	0.0272	40	30	0	0				
12	0.945	0.0273	15	15	0	0				
Total			405	405	423.0621	412.069				

be verified as shown in Table 7. However, by injecting 30 kVar at bus 10, the voltage profile of all the buses in the feeder improves and no bus voltage falls below the accepted range. This result is

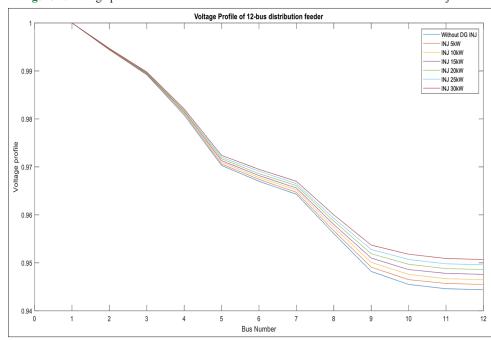
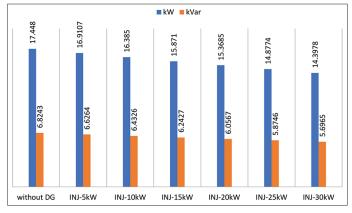


Figure 4: Voltage profile of the 12-bus radial distribution feeder with and without DG injection

Figure 5: Line losses in the 12-bus radial distribution feeder without DG injection and with different DG sizes



shown in Table 8. Furthermore, the total active loss improves by 17.48% and the total reactive loss also improves by 16.53%. This can be verified from Table 9. This further confirms that bus 10 is the optimal location for distributed generator installation as given by the ISNT approach. The following figures clearly give the curve of the voltage profile and the pictorial display of reduction in the line losses.

Figure 4 shows how the voltage profile of the base case load flow result relates to the other cases when different sizes of DGs (5 kW, 10 kW, 15 kW, 20 kW, 25 kW and 30 kW) are injected at the optimal location obtained through the IPSN approach. From the figure, it is clearly seen that with 30 kW DG, the voltage profile improves and falls within the acceptable range and set constraint. Also, Figure 5 shows a reduction in power loss by injecting different sizes of DG at the optimal location obtained through the ISNT approach.

Table 7: Radial 12-bus feeder total loss with distributed generator at bus 9

From To kW kVAr kW kVAr 1 0 0 0 2 500.1156 208.1909 2.6508 1.1035 2 60 60 60 1 -497.4648 -207.0874 2.6508 1.1035 3 424.7366 177.2127 2.0947 0.874 4 30 30 30 2.9463 1.2336 4 377.524 158.0712 2.9463 1.2336 5 5 5 5 5 3 -374.5777 -156.8376 2.9463 1.2336 5 307.9606 128.3813 3.0411 1.2678 5 307.9606 128.3813 3.0411 1.2678 6 268.6562 111.8377 0.809 0.3368 7 245.4755 102.159 0.6229 0.2592 7 55 55 5 6 -244.8526 -101.8997 0.6229	generator at bus 9									
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11 -17.2507 -4.8909 0.0036 0.001		12			0.0036	0.001				
	12									
Total 14.5529 5.7419		11	-17.2507	-4.8909						
	Total				14.5529	5.7419				

Table 8: Radial 12-bus network load flow result with distributed generator at bus 10

Bus number	Voltage magnitude	Voltage magnitude Angle degree		oad	Substation		Injected kVar
			kW	kVAr	kW	kVAr	
1	1	0	0	0	423.0621	412.069	0
2	0.9947	0.0023	60	60	0	0	0
3	0.9898	0.0044	40	30	0	0	0
4	0.9821	0.008	55	55	0	0	0
5	0.9724	0.0127	30	30	0	0	0
6	0.9695	0.0142	20	15	0	0	0
7	0.967	0.0155	55	55	0	0	0
8	0.96	0.0205	45	45	0	0	0
9	0.9537	0.0253	40	40	0	0	0
10	0.9518	0.0269	5	30	0	0	0
11	0.9509	0.0274	40	30	0	0	0
12	0.9507	0.0275	15	15	0	0	0
Total			405	405	423.0621	412.069	0

Table 9: Radial 12-bus feeder total loss with distributed generator at bus 10

From To kW kVAr kW kVAr 1 0 0 0 2 499.9421 208.1186 2.649 1.1027 2 60 60 60 1 -497.293 -207.0159 2.649 1.1027 3 424.565 177.1412 2.093 0.8733 4 30 30 2.9436 1.2325 4 377.3544 158.0002 2.9436 1.2325 4 55 55 55 3 -374.411 -156.7677 2.9436 1.2325 5 307.7939 128.3118 3.0377 1.2664 6 268.4929 111.7697 0.808 0.3364 6 268.4929 111.7697 0.808 0.3364 7 245.3134 102.0915 0.6221 0.2589 7 55 55 5 6 -244.691 -101.8326 0.6221 0.2589	generator at bus 10									
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4. CONCLUSION

This review was carried out basically to extensively itemize previous methodologies employed by various researchers to determine the optimal sites and optimal size of distributed generator. This was done under the impact of distributed generator in a network. These methodologies are summaries under three categories namely;

analytical, meta-heuristic and hybrid optimization methods. All the methods are based on solving non-linear load flow solution except the Inherent Structural Characteristic Indices. In this review work, inherent structural network topology is recommended for further studies to solve power system problems because of it is very easy to model compared to nonlinear and iterative approaches.

5. ACKNOWLEDGMENT

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