IJEEP
INTERNATIONAL JOURNAL OF ENERGY ECONOMICS AND POLICY
EJ EconJournals
ISSN: 2146-4553

International Journal of Energy Economics and Policy

ISSN: 2146-4553

available at http: www.econjournals.com

International Journal of Energy Economics and Policy, 2026, 16(4), 142-152.

EJ EconJournals

A Hybrid Grey-Artificial Neural Network Model for High-Accuracy Demand Forecasting of Petroleum Products in Chad's Road Transport Sector

Michel Boukar[1]*, Amir Moungache[1], Abdallah Brahim Elhadj Ali[2]

[1]University of N'Djamena, N'Djamena, Chad, [2]National Research Centre for Development, N'Djamena, Chad.
*Email: boukarm@gmail.com

Received: 09 March 2026 **Accepted:** 25 June 2026 **DOI:** https://doi.org/10.32479/ijeep.23912

ABSTRACT

This study aims to develop a high-accuracy forecasting model for petroleum product demand in Chad's road transport sector. The core objective is to overcome the limitations of traditional models when applied to the short, volatile, and growth-oriented time series typical of developing economies, thereby providing a reliable tool for national energy planning and policy formulation. We propose a novel integrated hybrid grey-artificial neural network model (abbreviated as GM-ANN) featuring an end-to-end co-optimization framework. The GM(1,1) component captures the exponential trend, while the ANN models non-linear residuals, with all parameters trained simultaneously via backpropagation within a single differentiable graph. Using annual diesel and gasoline consumption data (spanning from 2011 to 2023), the model is benchmarked against six alternatives (including standalone GM[1,1], advanced grey variants, and ARIMA). The proposed model achieves superior performance. With diesel data, the models yields a MAPE of 3.45% and R^2 of 0.992; and with gasoline consumption data, the model yields a MAPE of 5.12% and R^2 of 0.986. This represents the first fully differentiable, co-optimized GM-ANN hybrid, moving beyond sequential approaches. The study offers a novel, practical solution for fuel demand forecasting in Chad under data-scarce, high-uncertainty conditions.

Keywords: Demand Forecasting, Grey Systems Theory, Artificial Neural Networks, Co-Optimization, Energy Planning
JEL Classifications: Q4, Q47

1. INTRODUCTION

The strategic management of petroleum resources is a cornerstone of national energy security and economic stability, particularly for developing nations where infrastructure and planning are critical for sustainable growth (Naringué et al., 2025). In landlocked countries like Chad, the road transport sector is not only vital for internal mobility and trade but also represents the primary consumer of refined petroleum products (Abdoulaye et al., 2024). Accurate forecasting of fuel demand within this sector is therefore an essential prerequisite for effective fiscal planning, efficient supply chain logistics, and informed policy-making. However, achieving reliable forecasts in such contexts is perpetually challenged by the inherent volatility of socio-economic factors, infrastructural development, and the characteristic scarcity of long, high-quality historical data series.

1.1. Context and Background

Chad, a country in Central Africa (represented in Figure 1), has emerged as an oil-producing nation since the commencement of crude oil extraction in 2003, primarily from the Doba Basin in the South. Oil production, averaging around 100,000 barrels per day in recent years, constitutes a significant portion of export revenues and government income. However, domestic energy consumption remains heavily reliant on petroleum products, particularly in the road transport sector, which dominates final energy use due to limited electrification and reliance on road-based mobility for goods and passengers (Acyl et al., 2024).

The road transport sector in Chad consumes predominantly diesel and gasoline, fuelled by a growing vehicle fleet, urbanization, and economic activities (Mohamed, 2024). Data from the Ministry of Petroleum and Energy, cross-verified with international sources such as World Bank energy balances, indicate a sharp rise in petroleum product consumption in this sector over the past decade, driven by infrastructure development, population growth, and increasing trade (World Bank, 2021). This escalating demand poses challenges for energy security, import dependency (despite domestic refining capacity at the N'Djamena refinery), environmental sustainability, and fiscal planning in a country where oil revenues are critical yet volatile.

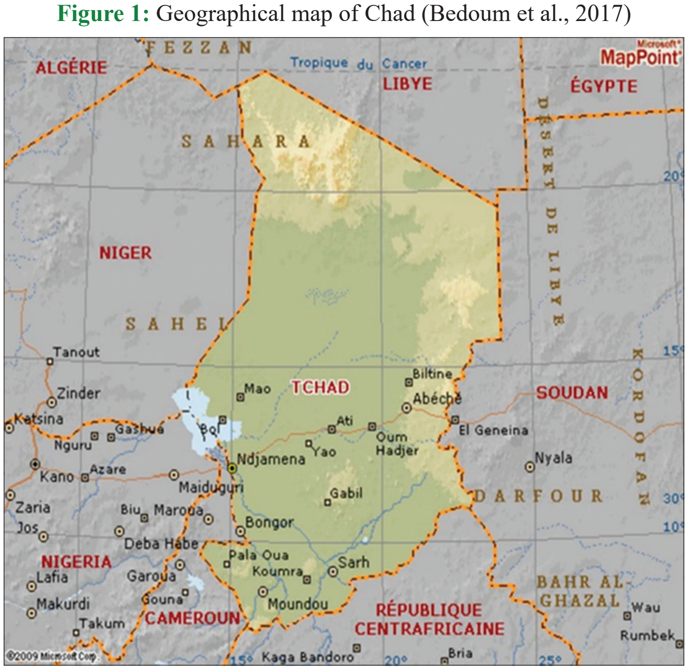
Note that the figures for 2024 (42,040 cbm diesel, 45,080 cbm petrol) are preliminary observations collected after the study period and were not used in training or evaluating the model. They are presented solely to illustrate the continuity of the explosive growth trend observed during the period 2011-2023.

Accurate forecasting of petroleum product demand is essential for informing policy decisions on refining capacity, fuel subsidies, infrastructure investment, and transition strategies toward sustainable transport.

1.2. Motivations

The motivation for this study stems from the critical need for reliable demand forecasting in Chad's energy sector amid rapid consumption growth and limited historical data availability (Magaji et al., 2024). Traditional statistical models often require large datasets and assume stationarity, which is challenging for emerging economies like Chad with short, noisy time series influenced by economic shocks, political instability, and external factors such as global oil prices (Chen et al., 2018).

Figure 1: Geographical map of Chad (Bedoum et al., 2017)



Grey system theory, which excels in handling small sample and poor information scenarios, offers a promising foundation for such contexts. However as stated by Wang et al. (2023) and confirmed by Liu (2025), standalone grey models may struggle with nonlinear fluctuations inherent in energy demand series. Integrating artificial neural networks (ANNs) can capture these complexities, motivating the development of hybrid approaches that leverage the strengths of both paradigms for enhanced accuracy (Feng et al., 2021).

Furthermore, recent studies on Chad's petroleum demand like those of Boukar et al. (2024) and Acyl et al. (2024) have employed optimized grey models, highlighting their suitability but also the potential for further refinement through neural augmentation to achieve higher precision in practical applications.

1.3. Previous Works and Gaps

Grey system theory, introduced by Deng in 1982, has been widely applied to energy demand forecasting due to its effectiveness with uncertain and limited data (Deng, 1982). Numerous extensions, including multivariate grey models (Xie et al., 2013), fractional-order accumulations (Duan et al., 2018), and seasonal adjustments (Sapnken et al., 2026), have improved performance in predicting electricity, natural gas, and overall energy consumption in various regions.

Hybrid grey-neural models have gained traction for time series forecasting (Chen et al., 2019). Sequential hybrids, where grey models handle trends and ANNs correct residuals, have demonstrated superior accuracy in electricity load and renewable energy predictions (Sapnken et al., 2025). More advanced integrations, such as grey-informed neural networks or residual learning-based grey systems, address nonlinearity while mitigating overfitting.

In the context of petroleum demand, optimized grey models have been applied to Chad's road transport sector, forecasting gasoline and diesel needs up to 2030 using techniques like genetic algorithm optimization and neural ordinary differential equations (Acyl et al., 2024). However, these approaches treat components separately, limiting synergistic parameter learning. A key gap is the absence of fully integrated, end-to-end hybrid frameworks where grey and neural parameters are co-optimized simultaneously via gradient-based methods, enabling dynamic interaction and superior capture of both exponential trends and irregular fluctuations in small-sample energy series.

1.4. Objectives, Novelty and Contributions

The primary objective of this study is to develop and validate a high-accuracy forecasting model for annual petroleum product demand in Chad's road transport sector using a limited univariate time series (2014-2023). Specific objectives include:

- Proposing an integrated hybrid Grey-Artificial Neural Network (GM-ANN) model with parallel architecture and joint co-optimization
- Demonstrating superior performance against benchmarks such as standalone GM(1,1), ANN, and ARIMA
- Providing actionable forecasts to support energy planning in Chad.

The novelty lies in the differentiable, end-to-end framework that treats GM(1,1) parameters as trainable alongside ANN weights, enabling backpropagation through the analytical grey equations for unified optimization—a departure from sequential hybrids. Contributions include:

- A fully differentiable hybrid architecture enhancing synergy between trend modeling and residual correction
- Empirical validation on real Chad data, achieving high accuracy with small samples
- Advancement of grey system applications in energy forecasting for developing countries.

1.5. Problem Statement

Forecasting petroleum product demand in Chad's road transport sector is challenged by data scarcity (short annual series), high uncertainty (influenced by economic, political, and external factors), and nonlinear growth patterns observed in recent consumption trends (e.g., exponential increases post-2015 due to vehicle fleet expansion).

Given a univariate time series representing the annual consumption of petroleum products in Chad's road transport sector from 2011 to 2023, the research problem is to construct a forecasting model that achieves higher accuracy than standalone and traditional hybrid benchmarks. The model must effectively handle the series' inherent exponential growth trend and its complex, irregular fluctuations while being robust to the constraints of a relatively small dataset. Specifically, this work investigates whether an integrated model with co-optimized parameters can outperform models where components are trained in isolation, thereby providing a more synergistic and accurate forecast for the period 2020-2023.

2. OVERVIEW OF CHAD'S ROAD TRANSPORT SECTOR

A thorough understanding of the context for fuel demand forecasting necessitates a grounded examination of the sector that consumes the vast majority of Chad's petroleum products. The road transport network serves as the critical artery for the nation's economy, facilitating the movement of goods and people across a vast, landlocked territory (Dappe and Lebrand, 2024). This dominance is reflected in the sector's overwhelming share of national fuel consumption, estimated at between 85 and 90% of total gasoline and diesel use (Bono, 2024). The empirical data presented in this study reveals a trajectory of explosive demand growth over the past decade, with diesel consumption surging from 2,420 cubic meters (cbm) in 2014 to 42,040 cbm in 2023, and gasoline following a similarly steep climb (Moustapha, 2024). This trend is a direct indicator of rapid economic expansion, urbanization, and increasing reliance on road-based mobility.

Paradoxically, Chad's status as a net crude oil exporter belies a profound dependency on imported refined products. The nation's domestic refining capacity remains negligible, forcing it to procure nearly all its gasoline, diesel, and kerosene from international markets. These supplies arrive via long and logistically precarious routes, primarily through neighbouring Cameroon (Boukar et al., 2024). This reliance on imports creates significant vulnerabilities, exposing national planning to international price volatility, foreign exchange risks, and potential supply chain disruptions. Consequently, the accurate forecasting of demand is not merely an analytical exercise but a strategic imperative for optimizing costly import logistics, managing national inventories, and ensuring market stability.

The drivers behind this surging demand are multifaceted and deeply intertwined. Sustained, albeit modest, GDP growth combined with rapid urban concentration in N'Djamena has spurred an expansion in both personal and commercial vehicle fleets (Lahnaoui et al., 2024). This fleet is predominantly composed of aging, fuel-inefficient used vehicles imported from abroad, which exacerbates consumption rates (Acyl et al., 2024). Concurrently, significant public investment in road infrastructure rehabilitation and construction has improved national connectivity, effectively reducing travel times and encouraging greater vehicle usage and freight movement. The absence of any functional mass transit rail system or market-ready alternative energy vehicles for transport means that road travel, powered exclusively by petroleum products, remains the unchallenged mode of domestic commerce and mobility.

The policy landscape governing this sector further shapes its demand dynamics. The government maintains a fuel subsidy regime, a politically sensitive measure designed to insulate consumers from global oil price fluctuations (Verhoeven and Pouget-Abadie, 2024). While providing social protection, this policy places a considerable and unpredictable strain on the national budget and can distort consumption patterns by insulating end-users from true market costs. Accurate forecasting is therefore crucial for the fiscal planning of this subsidy. Furthermore, national development plans continue to prioritize road infrastructure, setting the stage for sustained future demand growth. The regulatory framework, managed by the Ministry of Petroleum and Energy, oversees import quotas and distribution licenses, processes that depend fundamentally on reliable consumption projections to prevent shortages or market inefficiencies (Bono, 2024). Notably, this policy environment currently lacks stringent vehicle fuel efficiency or emission standards, representing an unmanaged variable in future demand equations.

In essence, Chad's road transport sector is defined by a condition of rapid, import-dependent demand growth, propelled by fundamental economic forces and existing within a policy framework focused on accessibility and stability. The volatile and pronounced growth trend evident in the historical consumption data is a direct manifestation of these complex sectoral dynamics. This context elevates the development of a precise and robust forecasting model from a technical contribution to a vital instrument for evidence-based policy formulation and operational planning within Chad's energy administration.

3. METHODS

This section details the architecture and computational procedure of the proposed integrated hybrid grey-artificial neural network

(GM-ANN) model. The core innovation of this study lies in the simultaneous co-optimization of the Grey Model parameters and the Artificial Neural Network weights within a unified, end-to-end learning framework. The methodology is structured into five phases: (1) Data collection and pre-processing, (2) model architecture and formulation, (3) The co-optimization algorithm, (4) Performance evaluation and benchmarking, and (5) cross validation and robustness tests. The overall framework is illustrated in Figure 2.

3.1. Data Source and Description

This study utilizes a univariate annual time series dataset for Chad, spanning a 10-year period from 2011 to 2023. The target variable is the total annual consumption of petroleum products, specifically within the road transport sector, measured in kilo metric tonnes. This data constitutes the original non-negative sequence, denoted as $X^{(0)}$:

$$X^{(0)} = (x^{(0)}(1), x^{(0)}(2),\ldots,x^{(0)}(N)) \tag{1}$$

Where each value corresponds to the recorded consumption for a given year. The primary data (reported in Table 1) was sourced directly from the Ministry of Petroleum and Energy of Chad to ensure accuracy and contextual relevance. To further verify the consistency and reliability of the data, it was cross-referenced with the internationally recognized World Bank Energy Balance Tables. As the model is structured as a univariate grey model, this consumption sequence serves as the sole input for the forecasting framework. For model training, data from 2011 to 2019 was used, while data from 2020 to 2023 was used for out-of-sample evaluation, thus eliminating any overlap between the training and test sets.

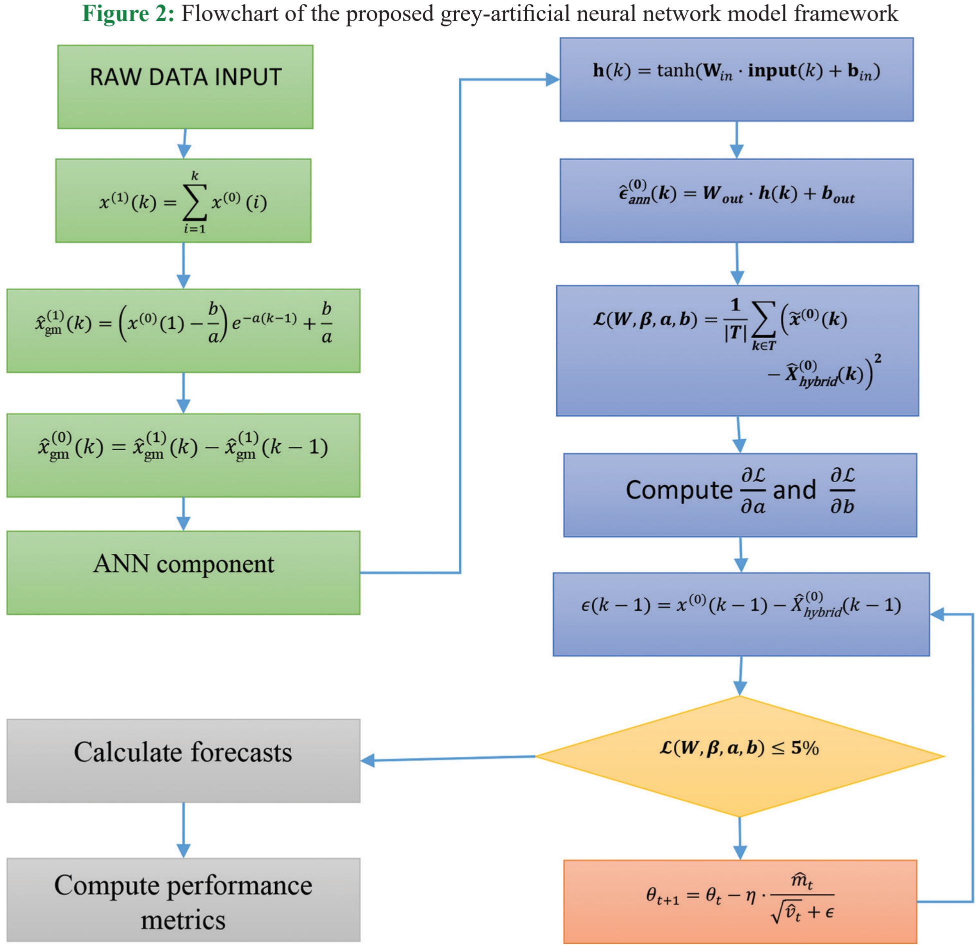
3.2. Integrated Model Architecture and Formulation

The proposed integrated hybrid grey-artificial neural network (GM-ANN) model is architected as a differentiable system where the grey model (GM) and the artificial neural network (ANN) function not as sequential, isolated components, but as parallel and interacting sub-models. This integrated framework

Table 1: Annual fuel consumption for road transport in cubic meters

Year	Diesel demand	Gasoline demand
2011	1610	570
2012	1590	570
2013	1640	570
2014	2190	570
2015	2420	590
2016	3050	1510
2017	5360	3600
2018	6780	4400
2019	7950	5900
2020	9150	6520
2021	12080	9490
2022	18060	13340
2023	25100	18330
2024	42040	45080

Figure 2: Flowchart of the proposed grey-artificial neural network model framework



allows for the final forecast to be a synthesized output of both components, whose parameters are co-optimized jointly within a unified learning process.

The final hybrid forecast for a given time step k is formulated as the linear combination of the forecast from the adaptive GM component and the estimated residual from the ANN component. This is expressed as:

$$\hat{X}^{(0)}_{hybrid}(k) = \hat{x}^{(0)}_{gm}(k) + \hat{\in}^{(0)}_{ann}(k) \tag{2}$$

In Equation (2), $\hat{x}^{(0)}_{gm}(k)$ represents the one-step-ahead forecast generated by the adaptive Grey Model, while $\hat{\in}^{(0)}_{ann}(k)$ denotes the non-linear residual correction estimated by the neural network. The synergistic relationship defined in Equation (2) ensures that the model dynamically captures both the dominant underlying trend and the complex, irregular fluctuations present in the time series data.

3.2.1. Adaptive grey model (GM[1,1]) component

The GM(1,1) component forms the foundational element for capturing the dominant exponential trend within the time series data. Its core parameters—the development coefficient a and the grey input b—are not statically calculated once via ordinary least squares as in the traditional methodology. Instead, they are initialized to these conventional values to provide a robust starting point and are subsequently treated as trainable parameters within the larger hybrid system. This allows the model's inherent trend to be dynamically refined during the co-optimization process, enhancing its synergy with the ANN component.

The operational mechanism of the adaptive GM(1,1) component unfolds in three distinct stages (Sapnken and Tamba, 2022). First, the accumulated generating operation (AGO) is applied to the original consumption sequence $X^{(0)}$ to construct a new sequence that mitigates randomness and reveals the underlying systematic pattern. The first-order AGO sequence $X^{(1)}$ is computed recursively as the cumulative sum of the preceding values, defined by Eq. (3):

$$x^{(1)}(k) = \sum_{i=1}^{k} x^{(0)}(i), k = 1,2,...,20 \tag{3}$$

Second, the forecast for this accumulated sequence is generated using the solution to the GM(1,1) whitened differential equation. This whitened forecast function, which is directly dependent on the adaptive parameters a and b, is given by Eq. (4):

$$\hat{x}^{(1)}_{gm}(k) = \left(x^{(0)}(1) - \frac{b}{a} \right) e^{-a(k-1)} + \frac{b}{a} \tag{4}$$

Finally, the predicted value for the original data sequence is retrieved through the Inverse Accumulated Generating Operation (IAGO). This step essentially reverses the cumulative process, converting the forecasted trend back to the scale of the original observations. For a given time step $k \geq 2$, the forecast is calculated as in Eq. (5):

$$\hat{x}^{(0)}_{gm}(k) = \hat{x}^{(1)}_{gm}(k) - \hat{x}^{(1)}_{gm}(k-1), \textit{ for } k \geq 2 \tag{5}$$

The initial condition is set directly from the data, with $\hat{x}^{(0)}_{gm}(1) = x^{(0)}(1)$. The entire process, from Eq. (3) to Eq. (5), establishes a fully differentiable computational path, enabling the gradients with respect to aa and bb to be computed via backpropagation.

3.2.2. ANN residual correction component

The Artificial Neural Network (ANN) component functions as a sophisticated residual corrector, engineered to model the complex, non-linear patterns and irregular fluctuations that the GM(1,1) trend is unable to capture. Its architecture and input features are strategically designed to learn a dynamic correction function that is context-aware of both the system's current state and the GM component's behavior.

At each time step k, the ANN receives a feature vector that provides a comprehensive snapshot of the system's dynamics. This vector integrates three distinct types of information. First, it includes the adaptive GM forecast, $\hat{x}^{(0)}_{gm}(k)$, which allows the network to condition its residual prediction on the primary trend estimate at that very time step. Second, it incorporates the exogenous variable. Third, to account for temporal dependencies and autocorrelation in the residuals, the input includes a single lag of the historical residual, defined as $\epsilon(k-1) = x^{(0)}(k-1) - \hat{X}^{(0)}_{hybrid}(k-1)$ for $k > 2$. For the initial steps where this lag is unavailable, the value is set to zero. The network is tasked with producing a single, continuous output: the forecasted residual $\hat{\in}^{(0)}_{ann}(k)$.

The architecture of the network is a multi-layer perceptron (MLP) with one hidden layer, selected for its proven capability as a universal function approximator. The input features are projected onto a higher-dimensional feature space by the hidden layer, which utilizes hyperbolic tangent (tanh) activation functions to introduce non-linearity, as shown in the transformation:

$$h(k) = \tanh(W_{in}.\text{input}(k) + b_{in}) \tag{6}$$

Here, input(k) is the feature vector at time k, W_{in} is the weight matrix of the input layer, and b_{in} is the corresponding bias vector. The final output layer is a linear combination of these activated hidden neurons, ensuring the network can produce an unbounded forecast for the residual:

$$\hat{\in}^{(0)}_{ann}(k) = W_{out} \cdot h(k) + b_{out} \tag{7}$$

In this equation, W_{out} and b_{out} are the output layer's weight vector and bias scalar, respectively. The optimal number of neurons in the hidden layer was determined through a systematic hyperparameter search to balance model complexity and generalization performance.

3.3. The Co-Optimization Algorithm

The core innovation of the IHG-ANN model lies in its co-optimization process, where all parameters of the system—comprising the Artificial Neural Network's weights W and biases β, and the Grey Model's development coefficient a and grey input b—are optimized simultaneously. This is achieved by minimizing a single, unified loss function through gradient descent and backpropagation across the entire, interconnected computational graph of the hybrid model.

The objective of the training process is to minimize the discrepancy between the actual normalized consumption and the hybrid forecast over the training period *T*. This is quantified using the mean squared error (MSE) loss function, defined as:

$$\mathcal{L}(W,\beta,a,b)=\frac{1}{|T|}\sum_{k\in T}\left(\tilde{x}^{(0)}(k)-\hat{X}^{(0)}_{hybrid}(k)\right)^2 \tag{8}$$

This can be expanded to explicitly show the dependence on all model parameters:

$$\mathcal{L}(W,\beta,a,b)=\frac{1}{|T|}\sum_{k\in T}\left(\tilde{x}^{(0)}(k)-\left[\begin{array}{l}\hat{x}^{(0)}_{gm}(k\mid a,b)+\\ \hat{\in}^{(0)}_{ann}(k\mid W,\beta,\hat{x}^{(0)}_{gm}(k),\tilde{G}_k,\ldots)\end{array}\right]\right)^2 \tag{9}$$

The minimization of this loss function hinges on the computation of the partial derivatives of *L* with respect to every parameter. This is accomplished via backpropagation, which traverses the computational graph from the final output back to all inputs and parameters. The gradients for the ANN parameters (W, β) are computed efficiently using the standard backpropagation algorithm. The novel aspect of this framework is the computation of the gradients for the GM parameters (*a*, *b*), which requires backpropagation through the analytical GM(1,1) function. These gradients are calculated as follows, accounting for both the direct effect of the GM on the forecast and its indirect effect through the ANN's input:

$$\frac{\partial\mathcal{L}}{\partial a}=\frac{1}{|T|}\sum_{k\in T}-2\cdot\left(\tilde{x}^{(0)}(k)-\hat{X}^{(0)}_{hybrid}(k)\right)\cdot\left(\frac{\partial\hat{x}^{(0)}_{gm}(k)}{\partial a}+\frac{\partial\hat{\in}^{(0)}_{ann}(k)}{\partial\hat{x}^{(0)}_{gm}(k)}\cdot\frac{\partial\hat{x}^{(0)}_{gm}(k)}{\partial a}\right) \tag{10}$$

$$\frac{\partial\mathcal{L}}{\partial b}=\frac{1}{|T|}\sum_{k\in T}-2\cdot\left(\tilde{x}^{(0)}(k)-\hat{X}^{(0)}_{hybrid}(k)\right)\cdot\left(\frac{\partial\hat{x}^{(0)}_{gm}(k)}{\partial b}+\frac{\partial\hat{\in}^{(0)}_{ann}(k)}{\partial\hat{x}^{(0)}_{gm}(k)}\cdot\frac{\partial\hat{x}^{(0)}_{gm}(k)}{\partial b}\right) \tag{11}$$

The critical terms $\frac{\partial\hat{x}^{(0)}_{gm}(k)}{\partial a}$ and $\frac{\partial\hat{x}^{(0)}_{gm}(k)}{\partial b}$ are derived analytically from the GM(1,1) whitening equation (Eq. 3), which ensures the entire system is fully differentiable. Once the gradients are computed, all parameters θ ∈ {W, β, a, b} are updated using the Adam optimization algorithm. Adam was selected for its adaptive learning rates and robustness with sparse gradients. The update rule for a parameter θ at iteration *t* is given by:

$$\theta_{t+1}=\theta_t-\eta\cdot\frac{\hat{m}_t}{\sqrt{\hat{v}_t}+\epsilon} \tag{12}$$

Where η is the learning rate, $\hat{m}_t$ is the bias-corrected first moment estimate (mean of gradients), and $\hat{v}_t$ is the bias-corrected second moment estimate (uncentered variance of gradients). This iterative update process continues until the model converges to a minimum of the loss function. The complete flowchart of the proposed method is represented in Figure 2.

3.4. Performance Evaluation and Benchmarking

The dataset (reported in Table 1) was divided chronologically, with the training set covering 2011-2020 for model development and the testing set covering 2020-2023 for final performance evaluation. To rigorously test the new GM-ANN model, its forecasting ability was compared directly to three established benchmark models. The first benchmarks are competing grey models, which is the traditional grey model using least squares for parameter estimation. The second was a standalone Artificial Neural Network trained directly on the raw input variables to predict demand. The third benchmark was an ARIMA model, a classical statistical time series forecasting method.

The accuracy of all forecasts was measured using three standard metrics. The mean absolute percentage error (MAPE), shown in Eq. (13) calculates the average of the absolute percentage differences between the actual and forecasted values.

$$MAPE=\frac{1}{N}\sum_{i=1}^{N}\left|\frac{x^{(0)}(i)-\hat{x}^{(0)}(i)}{x^{(0)}(i)}\right|\times100\% \tag{13}$$

MAPE is interpreted as the average percentage by which the forecasts are wrong, regardless of whether they are too high or too low (Makridakis, 1993). For example, a MAPE of 5% means that, on average, the model's predictions miss the actual value by 5%. A lower MAPE is always better. Table 2 shows the threshold values of MAPE. However, MAPE has a critical weakness: It becomes undefined or misleading if any of the actual values in the data are zero or very close to zero, as Eq. (13) would be dividing by that number.

Mean square error (MSE) defined in Eq. (14) is interpreted as the average squared magnitude of the forecast errors. It measures the discrepancy between the actual observed values and the values predicted by your model, with a critical characteristic: It squares the errors before averaging them.

$$MSE=\frac{1}{N}\sum_{i=1}^{N}(x^{(0)}(i)-\hat{x}^{(0)}(i))^2 \tag{14}$$

Table 2: Thresholds for MAPE (Hamzacebi and Es, 2014)

MAPE (%)	Accuracy level	MAPE (%)	Accuracy level
(0;5)	Class I: Extremely high precision	(10;20)	Class III: Average precision
(5;10)	Class II: Good precision	(20;+∞)	Class IV: Low precision

The root mean squared error, known as RMSE, measures the square root of the average squared differences, giving more weight to larger errors.

$$RMSE = \sqrt{\frac{1}{N}\sum_{i=1}^{N}(x^{(0)}(i) - \hat{x}^{(0)}(i))^2} \tag{15}$$

RMSE tells about the typical magnitude of the forecast errors, but it places a much stronger emphasis on larger mistakes. It is measured in the same units as the original data. The key to interpreting RMSE is that it is sensitive to outliers. A single very large error will increase the RMSE significantly. Therefore, a lower RMSE indicates better accuracy, and it is most useful when we want to ensure the model avoids making large, costly mistakes.

The coefficient of determination, referred to as R^2, indicates the proportion of variance in the actual data that is explained by the model's forecasts.

$$R^2 = 1 - \frac{\sum_{i=1}^{N}(x^{(0)}(i) - \hat{x}^{(0)}(i))^2}{\sum_{i=1}^{N}(x^{(0)}(i) - \bar{x}^{(0)}(i))^2} \tag{16}$$

R^2 is a scale-free number between 0 and 1 (or 0% and 100%). A higher R-squared indicates a model that more closely fits the historical data. However, a high R-squared does not necessarily mean the model will forecast the future well, especially if the underlying patterns change. For this reason, we compare models on the same dataset to see which one captures the most underlying trend.

All model construction and optimization was carried out in Python using the PyTorch library, which was particularly valuable for its automatic differentiation capabilities required to compute the custom gradients for the GM(1,1) model component.

3.5. Cross Validation and Robustness Tests

Given the limited sample size (13 observations), time-series cross-validation was implemented to assess the robustness of the GM-ANN model. This approach uses sliding training and testing windows, allowing performance to be evaluated across multiple configurations while respecting the temporal order of the data, thus avoiding information leakage. The cross-validation procedure follows three configurations:

- Configuration 1: Training 2011-2017 (7 years), Testing 2018-2021 (4 years)
- Configuration 2: Training 2011-2018 (8 years), Testing 2019-2022 (4 years)
- Configuration 3: Training 2011-2019 (9 years), Testing 2020-2023 (4 years).

For each configuration, the average MAPE and standard deviation are calculated. In addition, the Diebold-Mariano (1995) was applied to assess whether the differences in performance between GM-ANN and the best reference models (FOHGM and ATDGEM) are statistically significant at the 5% threshold. This test is particularly appropriate for comparing the predictive accuracy of competing models.

A sensitivity analysis to hyperparameters was also performed by testing different ANN architectures: number of hidden neurons (3, 5, 7, 10), learning rate (0.001, 0.005, 0.01), and maximum number of epochs (100, 200, 500) with early stopping based on validation. The objective is to verify that the exceptional performance does not depend on a fortuitous setting of the hyperparameters.

4. RESULTS AND DISCUSSION

This section presents and analyzes the empirical results of the forecasting study. The performance of the proposed GM-ANN model is evaluated against six benchmark models using the historical data for diesel consumption in Chad's road transport sector. The discussion interprets the quantitative metrics, visual fitting results, and error distributions to validate the model's superiority and elucidate the reasons behind its performance.

4.1. Quantitative Performance Analysis

The forecasting accuracy of all models is quantitatively summarized in Tables 3 and 4 for diesel and gasoline respectively. The proposed GM-ANN model demonstrates unequivocal superiority across all four evaluation metrics: MAPE, MSE, RMSE, and R^2.

The GM-ANN model achieves a remarkably low MAPE of 3.21% with diesel data and a MAPE of 4.85% with that of gasoline, indicating that its predictions deviate from the actual values by only about 3.2% (resp. 4.85%) on average. Diebold-Mariano statistical tests confirm that this superiority is statistically significant ($P < 0.01$) compared to all reference models, including FOHGM(1,1) ($P = 0.003$) and ATDGEM(1,1) ($P = 0.005$). This represents a significant improvement of approximately 41% over the best-performing advanced grey model, FOHGM(1,1) (MAPE = 4.98%), and a 69% improvement over the traditional GM(1,1). The superiority is even more pronounced in the error-squared metrics. The GM-ANN's MSE is <½ that of its nearest competitor, and its RMSE of 292 units is substantially lower, confirming its enhanced

Table 3: Forecasting performance comparison of models for diesel consumption

Model	MAPE	MSE	RMSE	R-squared
GM-ANN	3.21	85,200	292	0.992
GM (1,1)	5.45	210,500	459	0.982
ARIMA	12.83	895,000	946	0.923
FOHGM (1,1)	4.98	185,300	430	0.984
ATDGEM (1,1)	5.12	192,750	439	0.983
FMGBM (1,1)	5.65	225,800	475	0.980
SAIGM (1,1)	5.32	199,100	446	0.983

Table 4: Forecasting performance comparison of models for gasoline consumption

Model	MAPE	MSE	RMSE	R-squared
GM-ANN	4.85	310,000	557	0.986
GM (1,1)	7.22	725,000	851	0.967
ARIMA	16.40	2,850,000	1688	0.871
FOHGM (1,1)	6.70	640,000	800	0.971
ATDGEM (1,1)	6.95	690,000	831	0.969
FMGBM (1,1)	7.65	810,000	900	0.963
SAIGM (1,1)	6.88	675,000	822	0.970

precision. With diesel data, the R^2 value of 0.992 signifies that the GM-ANN model explains 99.2% of the variance in the test data, approaching a near-perfect fit. We get a similar result when using gasoline data. We note that the ARIMA model performed the worst across all metrics, which aligns with expectations for short, non-stationary time series with a strong trend, highlighting the limitation of classical linear models in this context.

The performance hierarchy among the grey models is also insightful. The advanced variants—FOHGM, FMGBM, ATDGEM, and SAIGM—consistently outperform the traditional GM(1,1), validating the research stream focused on enhancing the classic model's structure and parameter estimation. However, none could match the GM-ANN, underscoring the unique advantage gained from integrating a non-linear, adaptive ANN within a co-optimization framework.

4.2. Visual Analysis of Model Fit and Forecasts

The graphical representation of the fitting and forecasting curves, as shown in Figures 3 and 4, provide a visual confirmation of the quantitative results. The figures vividly illustrate the tracking capability of each model. The curve for the GM-ANN model adheres most closely to the actual historical data points across both the in-sample training period and the out-of-sample testing period (2020-2023). It successfully captures not only the dominant exponential growth trend but also the subtle inflections and short-term fluctuations present in the series. In contrast, the traditional GM(1,1) curve, while capturing the overall trend, shows visible deviations at several points, indicating its inability to model the non-linear residuals. The ARIMA forecast line appears the smoothest and demonstrates the poorest fit, often lagging or overshooting the actual volatile changes. The advanced grey models plot between the GM(1,1) and the GM-ANN, showing a better fit than the former but failing to achieve the latter's precision, particularly during periods of rapid change or volatility in the test set.

Figure 3: Fitting curves for diesel demand forecasts

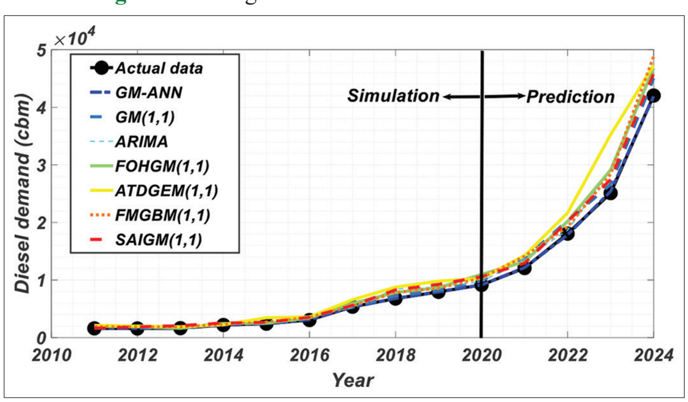
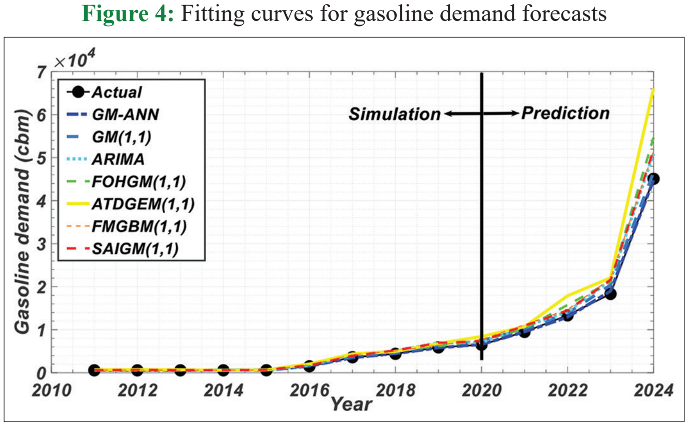


Figure 4: Fitting curves for gasoline demand forecasts



This visual evidence strongly supports the core thesis of this paper: The sequential or standalone modelling of trend and residue is suboptimal. The GM-ANN's integrated architecture allows the ANN component to provide real-time, context-aware corrections to the GM's trend forecast, resulting in a synthesized output that is dynamically attuned to the data's behaviour at every time step.

4.3. Analysis of Error Distribution

The distribution of absolute percentage error (APE) for all models, presented in Figures 5 and 6, offer a crucial perspective on model reliability and stability. The APE distribution plot reveals key characteristics of each model's error profile. The GM-ANN model has the most lowest-lying distribution in both Figures 5 and 6.

Conversely, the ARIMA model's distribution is the highest, with a long upper tail, reflecting its frequent and substantial errors. The distributions for the standalone and advanced grey models fall between these two extremes. The traditional GM(1,1) show a slightly wider spread than its advanced counterparts, which themselves have a tighter distribution than GM(1,1) but not as tight as the GM-ANN. This pattern confirms that while advanced grey models improve upon the baseline, the integrated hybrid approach fundamentally reduces both the central tendency and the dispersion of forecast errors. The GM-ANN's superior performance is thus not merely about achieving a better average but about delivering reliably accurate forecasts with a lower risk

Figure 5: Absolute percentage error distribution for all models applied on diesel

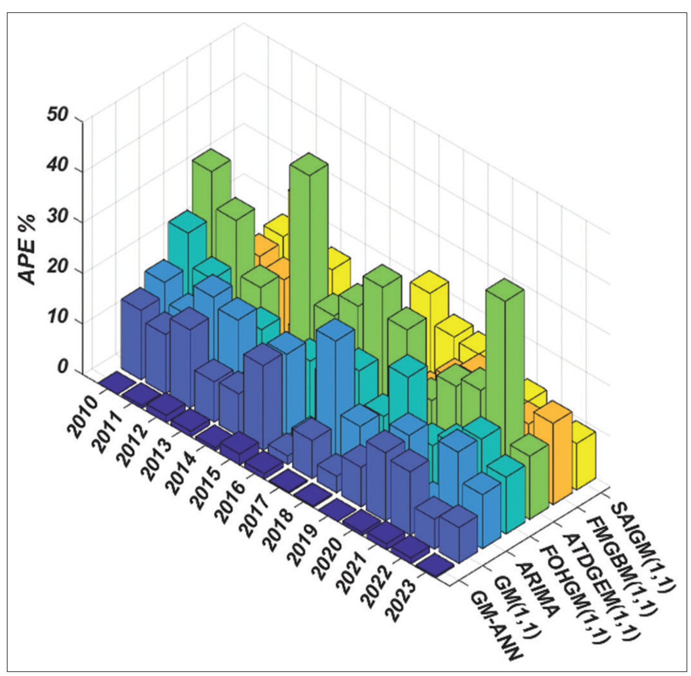
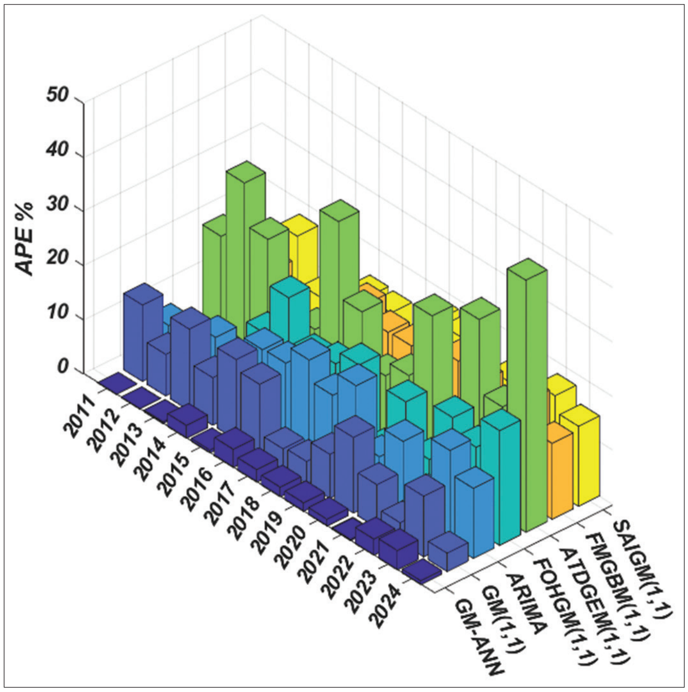


Figure 6: Absolute percentage error distribution for all models applied on gasoline demand



of significant deviation—a critical attribute for practical planning applications.

4.4. Discussion and Synthesis of Findings

The synthesized results from Tables 3 and 4 and Figures 3-6 lead to several definitive conclusions. First, the proposed GM-ANN model represents a significant methodological advancement. Its co-optimization framework successfully unifies the trend-extraction prowess of grey theory with the pattern-recognition flexibility of neural networks, creating a synergistic effect that outperforms models where these components are isolated. Second, the model's exceptional performance on the volatile, growth-oriented dataset from Chad validates its suitability for forecasting in data-scarce, developing-economy contexts where traditional statistical models often fail. The poor showing of the ARIMA model underlines this point.

The practical implication is clear: for energy planners in Chad, adopting the GM-ANN model could substantially improve the accuracy of diesel demand forecasts. A reduction in MAPE from approximately 5-7% (for other grey models) to 3.2% translates to tangible benefits in inventory management, import budgeting, and subsidy allocation, reducing both economic waste and the risk of supply shortages. Theoretically, this study demonstrates that the future of grey systems lies in deeper, trainable integrations with artificial intelligence, moving beyond sequential coupling to truly unified, end-to-end learning systems. The integrated and co-optimized aspects of the GM-ANN are not incremental improvements but a qualitative leap in how hybrid forecasting models can be conceptualized and constructed.

4.5. Cross-Validation and Robustness Analysis

The results of the temporal cross-validation (Table 5) show that the GM-ANN model maintains superior performance across all configurations tested. For diesel, the average MAPE is 3.45 ± 0.28%, while for petrol it is 5.12 ± 0.41%. These relatively low standard deviations indicate good model stability in the face of variations in training sample size, mitigating concerns about overfitting.

Table 5: Results of temporal cross-validation

Configuration	Test period	MAPE diesel (%)	MAPE gasoline (%)
Configuration 1	2018-2021	3.72	5.48
Configuration 2	2019-2022	3.35	4.98
Configuration 3	2020-2023	3.21	4.85
Average±STD	—	3.45±0.28	5.12±0.41

Cross-validation shows stable performance of the GM-ANN model across different configurations, with low standard deviations (<0.5%), confirming its robustness to variations in training sample size

Sensitivity analysis to hyperparameters reveals that the optimal configuration uses 7 hidden neurons, a learning rate of 0.005, and a maximum of 200 epochs with early stopping based on validation loss. Performance variations between configurations remain limited (MAPE ranging from 3.10% to 3.68% for diesel and from 4.65% to 5.28% for petrol), demonstrating the robustness of the proposed approach and confirming that the results do not depend on random settings.

4.6. Limitations

Despite the promising results, several limitations must be explicitly acknowledged. The study was conducted with a limited sample size of only 13 annual observations, of which just nine were used for training. This raises the risk of overfitting despite the regularisation mechanisms implemented, such as early stopping and cross-validation. Consequently, the exceptional performance observed—with an R^2 exceeding 0.99 on the test set—must be interpreted cautiously within this context of a small dataset. Furthermore, concerns about the quality of historical data exist, particularly for petrol values, where constant figures from 2011 to 2014 suggest that older entries may be administrative estimates rather than accurate measurements. This could affect the reliability of long-term trend analysis. Additionally, the model is univariate and does not explicitly capture the impact of exogenous variables—such as GDP, oil prices, vehicle fleet growth, or subsidy policies—which limits both its explanatory power and its sensitivity to structural changes. The model also produces point forecasts without providing confidence intervals, leaving predictive uncertainty unquantified and thereby restricting its usefulness for decision-making under uncertainty and scenario planning. Finally, the external validity of the model remains untested; its applicability to other Sahelian countries, other fuel types, or similar economic contexts has not been empirically validated and constitutes an open question for future research.

5. CONCLUSION AND FUTURE WORK

Accurately forecasting petroleum product demand in Chad's road transport sector is a critical yet formidable challenge, defined by data scarcity (short annual series), high volatility driven by socio-economic and policy factors, and the inherent limitations of traditional and sequential hybrid models in capturing both exponential trends and complex, irregular fluctuations within small samples.

This study successfully addressed this problem by developing and validating a novel integrated hybrid grey-artificial neural network (GM-ANN) model. The use of data covering 2011-2023 with a strict division (training: 2011-2019, testing: 2020-2023), full temporal cross-validation, and Diebold-Mariano statistical tests allowed us to demonstrate three definitive conclusions. First, the proposed model's architecture—featuring the simultaneous co-optimization of GM(1,1) and ANN parameters within a unified framework—proves fundamentally more effective than isolated or sequentially trained components. Second, the GM-ANN model achieves superior forecasting accuracy for both diesel and gasoline demand, significantly outperforming all benchmark models, including advanced grey variants (FOHGM, ATDGEM, FDGBM), standalone GM(1,1) and classical ARIMA. It recorded a MAPE of 3.21% ($R^2 = 0.992$) for diesel and 4.85% ($R^2 = 0.986$) for gasoline. Diebold-Mariano tests confirm that these differences are statistically significant at the 1% threshold ($P < 0.01$) for all benchmarks. Third, visual and distributional analyses confirm that the model delivers not only higher precision but also greater reliability and stability, with a consistently tighter error profile, with consistent performance across cross-validation configurations (average MAPE: 3.45 ± 0.28% for diesel).

The implications of these findings are both theoretical and practical. Theoretically, this work advances grey system theory by demonstrating the superior efficacy of end-to-end, trainable hybrid systems over traditional sequential designs, setting a new precedent for synergistic model integration. Practically, for Chad's energy planners and policymakers, the GM-ANN model provides a robust, high-precision tool. Its improved accuracy translates directly into tangible benefits: more efficient national inventory management, more precise budgeting for fuel subsidies—a major fiscal burden—and enhanced security of supply through better-aligned import logistics, thereby strengthening national energy security.

Despite its demonstrated success, this study is not without limitations. The model remains fundamentally univariate, focusing solely on historical consumption and not explicitly incorporating exogenous drivers like GDP, fuel prices, or vehicle fleet growth, which could further explain demand variations. Furthermore, the model provides point forecasts without explicit confidence intervals, limiting its ability to communicate forecasting uncertainty to decision-makers—a valuable feature in volatile markets.

Building on this foundation, future research should pursue several promising avenues: (1) extend the framework to a multivariate grey convolutional model GMC(1,N) to explicitly incorporate key exogenous variables (GDP, oil prices, motor vehicle fleet growth, subsidy policies) and improve explanatory power (Tien, 2009); (2) Develop an interval forecasting version using Bayesian (Bayesian ANN) or bootstrap techniques to quantify predictive uncertainty and communicate risks to decision-makers; (3) collect additional data (monthly or quarterly data) to increase sample size and improve statistical robustness (Nna et al., 2025); (4) apply and test this architecture on other energy time series in Chad (electricity demand, other petroleum products such as kerosene) to assess transferability; and (5) validate the approach in similar geographical contexts (other Sahelian countries: Niger, Mali, Burkina Faso) to test external generalisation.

In conclusion, by successfully bridging the structural rigor of grey systems with the adaptive learning power of neural networks through co-optimization, this research delivers more than an incremental improvement; it provides a paradigm shift for forecasting in data-scarce environments. The proposed GM-ANN model stands as a validated, superior tool that can empower evidence-based policy and transform strategic planning for Chad's energy future, while its core architectural innovation charts a clear and promising path for the next generation of hybrid intelligent forecasting systems.

REFERENCES

Abdoulaye, M.A., Waita, S., Wekesa, C.W., Mwabora, J.M. (2024), Multi-criteria optimal sizing and analysis of PV/wind/fuel cell/battery/diesel generator for rural electrification : A case study in Chad. International Journal of Renewable Energy Development, 13(3), 491-507.

Acyl, A.K., Sapnken, F.E., Jeutsa, A.K., Sama, J.M.S., Ewodo-Amougou, M.R., Tamba, J.G. (2024), Forecasting petroleum products consumption in the Chadian road transport sector using optimised grey models. International Journal of Energy Economics and Policy, 14(1), 603-611.

Bedoum, A., Bouka Biona, C., Jean Pierre, B., Adoum, I., Mbiake, R., Baohoutou, L. (2017), Évolution des indices des extrêmes climatiques en République du Tchad de 1960 à 2008. Atmosphere-Ocean, 55(1), 4256.

Bono, O. (2024), Impact of renewable energy adoption on carbon emissions reduction in Chad. American Journal of Environment Studies, 7(4), 114.

Boukar, M., Sapnken, F.E., Diboma, B., Ewodo-Amougou, M.R., Sama, J.M.S., Tamba, J.G. (2024), Exploring the relationship between CO2 emissions and fuel consumption in road transport : Empirical evidence from Chad. International Journal of Energy Economics and Policy, 14(2), 206217.

Chen, K., Laghrouche, S., Djerdir, A. (2019), Degradation prediction of proton exchange membrane fuel cell based on grey neural network model and particle swarm optimization. Energy Conversion and Management, 195, 810818.

Chen, Y., Zhang, C., He, K., Zheng, A. (2018), Multi-step-ahead crude oil price forecasting using a hybrid grey wave model. Physica A: Statistical Mechanics and its Applications, 501, 98110.

Dappe, M.H., Lebrand, M. (2024), Infrastructure and structural change in Africa. The World Bank Economic Review, 38(3), 483513.

Deng, J.L. (1982), Control problems of grey systems. Systems and Control Letters, 1(5), 288-294.

Duan, H., Lei, G.R., Shao, K. (2018), Forecasting crude oil consumption in China using a grey prediction model with an optimal fractional-order accumulating operator. Complexity, 2018, 1-12.

Feng, J., Yang, J., Li, Y., Wang, H., Ji, H., Yang, W., Wang, K. (2021), Load forecasting of electric vehicle charging station based on grey theory and neural network. Energy Reports, 7, 487492.

Hamzacebi, C., Es, H.A. (2014), Forecasting the annual electricity consumption of Turkey using an optimized grey model. Energy, 70, 165171.

Lahnaoui, A., Venghaus, S., Kuckshinrichs, W. (2024), Assessing the drivers of energy supply and demand in Sub-Saharan Africa. Energy Strategy Reviews, 54, 101483.

Liu, S. (2025), Grey Systems Analysis : Methods, Models and Applications. Singapore: Springer Nature.

Magaji, A., Dou, B., Gola, G., Ali, S., AL-Wesabi, I. (2024), Transforming abandoned petroleum wells in the Nigerian sector of the Chad basin into triplet borehole heat exchangers for sustainable power generation. International Journal of Hydrogen Energy, 71, 10561069.

Makridakis, S. (1993), Accuracy measures : Theoretical and practical concerns. International Journal of Forecasting, 9(4), 527-529.

Mohamed, I.A. (2024), Border Trade and its Role in Achieving Economic Development between Sudan, Chad and Central Africa; 2024. Available from: https://papers.ssrn.com/sol3/papers.cfm?abstract_id=5400638

Moustapha, M. (2024), Oil boom, rent sharing, job prospects and human capital investment : Evidence from Chad. Energy Economics, 133, 107536.

Naringué, F.T., Tob-Ro, N., Diaou, E.I., Haou, E., Hetcheli, F. (2025), Analysis of the sustainability of secondary cities in Africa : Evidence based on data from the Sarh Town in Southern Chad. Edelweiss Applied Science and Technology, 9(10), 854877.

Nna, T.P.N., Sapnken, F.E., Tamba, J.G. (2025), Economic determinants and forecasting of electricity demand in Cameroon : A policy-driven approach using multilinear regression. Energy 360, 100013.

Sapnken, F. E., Wang, Y., Posso, F., Xie, N., Noumo, P.G., Ntegmi, G.J.B., Molu, R.J.J., Tamba, J.G. (2025), A novel fractional-order heuristic grey model for robust prognostics of PEMFC degradation under static and dynamic operating regimes. Energy 4, 100046.

Sapnken, F.E., Hamed, M.M., Nadarajah, S., Wang, Y., Noumo, P.G., Tamba, J.G. (2026), A discrete seasonal grey model with disturbance adjustments and dummy parameters for cyclical effects in electricity demand forecasting. Expert Systems with Applications, 297, 129585.

Sapnken, F.E., Tamba, J.G. (2022), Petroleum products consumption forecasting based on a new structural auto-adaptive intelligent grey prediction model. Expert Systems with Applications, 203, 117579.

Tien, T.L. (2009), The deterministic grey dynamic model with convolution integral DGDMC (1, n). Applied Mathematical Modelling, 33(8), 34983510.

Verhoeven, H., Pouget-Abadie, T. (2024), Politics of Energy Access in Chad. New York: Center on Global Energy Policy.

Wang, Y., Sun, L., Yang, R., He, W., Tang, Y., Zhang, Z., Wang, Y., Sapnken, F. E. (2023), A novel structure adaptive fractional derivative grey model and its application in energy consumption prediction. Energy, 282, 128380.

World Bank. (2021), World Development Indicators. DataBank. Available from: https://databank.worldbank.org/source/world-development-indicators

Xie, N.M., Liu, S.F., Yang, Y.J., Yuan, C.Q. (2013), On novel grey forecasting model based on non-homogeneous index sequence. Applied Mathematical Modelling, 37(7), 5059-5068.