



Enhancing Forecast Accuracy of Exchange Rate Volatility Using Hybrid ANN-GARCH Models: Evidence from South Africa, Brazil, and China

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ABSTRACT

An important development in the modelling of exchange rate volatility is the use of artificial neural networks (ANN) to create enhanced generalised autoregressive conditional heteroskedasticity (GARCH) models. Conventional GARCH models are good at capturing the clustering of volatility in financial time series, but they have trouble understanding complex linkages and non-linear patterns in the data. This paper aims to investigate the hybrid approach in modelling exchange rate volatility of two currency pairs: South African Rand against Brazilian Real (ZAR/REAL) and South African Rand against Chinese Yuan (ZAR/YUAN) using monthly observations over the period of January 1996-March 2024. The paper introduced ANN as an additional factor to both symmetric and asymmetric GARCH models that capture most common stylised facts about exchange rate volatility and leverage effects. The GARCH (1,1)-ANN, EGARCH (1,1)-ANN, and GJR-GARCH (1,1)-ANN models were used, and their performance was assessed using mean squared error (MSE), mean absolute error (MAE), and mean absolute percentage error (MAPE). The results revealed that the EGARCH (1,1)-ANN model had the best overall performance when compared to the other hybrid models based on all three evaluation measures for both the currency pairs' data. The paper recommends further similar studies to predict future exchange rate trends and also incorporating other nonlinear methods.

Keywords: ANN, EGARCH-ANN, Exchange Rate, GJR-GARCH-ANN, Heteroskedasticity, Volatility

JEL Classifications: C22, C53, F31, G17, G15

1. INTRODUCTION

This paper investigated the generalized autoregressive conditional heteroskedastic (GARCH) approach hybrid with artificial neural network (ANN) in modelling exchange rate volatility of two currency pairs. Accurately forecasting the volatility of currency rates is crucial for risk management and informed decision-making in the dynamic financial markets. Conventional econometric models, such as GARCH models, have been used for a long time to describe the time-varying features of exchange rate volatility. It is commonly recognised that GARCH models may accurately represent volatility clustering, which is characterised by high volatility intervals followed by additional high volatility intervals and vice versa. Despite their many benefits, GARCH models often

fail to reflect the complex patterns and nonlinear relationships observed in financial time series. The development of ANNs offers a workable solution to these problems. An increasing number of finance-related domains, including volatility forecasting, are using ANNs due to their ability to represent intricate and nonlinear interactions.

The GARCH statistical model, an extension of the ARCH model developed by Engle and generalised by Bollerslev (1986) along with the exponential GARCH (EGARCH) model that was introduced by Nelson (1991) and the Glosten-Jagannathan-Runkle GARCH (GJR-GARCH) model that developed by Glosten et al. (1993), were utilised for analysing time series data and assumes heteroskedastic variance error (Rasure, 2021). It has been

demonstrated that GARCH models may effectively predict future volatility of various financial time series and simulate conditional volatility (Goodwin, 2012).

The ANN model developed by Krenker et al. (2011) is a mathematical model that attempts to replicate the composition and operations of biological NNs. Pradhan and Kumar's (2010) paper went on to describe ANN as an information processing technique used to represent the mathematical connections between input and output variables. It is a member of the family of generalised nonlinear non-parametric models that arose from studies on the nervous system and the brain. ANN is used due to its reliable approaches for pattern categorisation, estimating continuous variables, and forecasting time series due to their adaptability as function approximators (Kaastra and Boyd, 1996).

Due to the limitations of existing methods, this paper suggests a hybrid approach that combines GARCH-type models with ANN. The GARCH-type models fail to capture the complex patterns, and nonlinear relationships typically present in financial time series, while the ANN model struggles to handle both nonlinear and linear patterns effectively on its own. The proposed hybrid approach aims to improve the accuracy of time series forecasts. Therefore, the focus of the paper is to enhance the modelling of exchange rate volatility through the integration of GARCH-type models with ANNs. The main research question of the paper is: "How does the integration of GARCH-type models with ANNs enhance the modelling and prediction of exchange rate volatility?" The enhanced GARCH-ANN model aims to improve forecast accuracy and capture more subtle patterns in volatility dynamics by fusing the resilience of GARCH models with the flexibility and adaptive learning capabilities of ANNs. The combination allows the researcher to model both the linear and non-linear aspects of the data. The structure of the paper is as follows: Section 2 offers a review of the literature, Section 3 outlines the methodology, Section 4 covers the data analysis and interpretation of the results, and Section 5 concludes the paper.

2. LITERATURE REVIEW

The paper examines relevant empirical research that was carried out with GARCH-ANN hybrid models. Several studies have found significant benefits from combining ANN with GARCH-type models in the form of a hybrid model. Studies such as that of Lu et al. (2023) were conducted to model the volatility of the Shanghai stock price index using hybrid ANN and GARCH-type models. It was discovered that the EGARCH-ANN hybrid model outperformed other models when modelling the volatility of the log-return value of stocks on the Shanghai stock market.

Exchange rate volatility has been the subject of numerous studies in the past, including those by Kristjanpoller and Minutolo (2015) and Lahmiri (2017). This is due to exchange rate fluctuations, making it a significant component of the FOREX market that affects economies globally. Several GARCH-ANN hybrid models were used in these studies. However, this paper makes use of enhanced general autoregressive conditional heteroskedasticity (GARCH (1,1)-ANN, EGARCH (1,1)-ANN, and GJR-GARCH

(1,1)-ANN) models to estimate the volatility of the ZAR/REAL and ZAR/YUAN exchange rates.

Due to the GARCH model's symmetric nature, which made it difficult for it to capture the leverage effect in the data, the EGARCH and GJR-GARCH models were introduced. Because these models are asymmetrical, they are able to overcome these shortcomings (Liu and So, 2020). To handle the high volatility that exchange rate data contains, uncover hidden patterns within the data, and account for the nonlinear effect of volatility that the GARCH type models might miss, ANN is also introduced, to enhance the accuracy of GARCH type models in modelling exchange rate volatility. The nonlinear effects of volatility that are not simulated by GARCH type models can be simulated using neural network (NN) models. This was initially shown by Donaldson and Kamstra (1997), who developed a semi-nonparametric nonlinear GARCH model based on the ANN literature and tested its ability to predict stock return volatility in London, New York, Tokyo, and Toronto.

Kristjanpoller and Minutolo (2015) expanded the realm of expert systems, forecasting, and modelling by combining ANN with the GARCH technique, resulting in an ANN-GARCH. The hybrid ANN-GARCH model forecasts gold price volatility in the present and future. The findings demonstrated that employing the ANN-GARCH strategy rather than the GARCH method alone improved prediction in general. ANN-GARCH enabled a 25% decrease in the mean average percent error overall. The findings were derived using the Euro/Dollar and Yen/Dollar exchange rates, the DJI and FTSE stock market indexes, and the oil price return as inputs.

Hajizadeh et al. (2012) used two hybrid NN models to enhance the ability of GARCH models to forecast the return volatility of the S&P 500 index. The study proposed two hybrid models based on EGARCH and ANN, the results saw the hybrid model outperforming the GARCH method. Bildirici and Ersin (2009) used an NN model in conjunction with several GARCH models using the Istanbul Stock Exchange, identifying improvements in RMSE for most of the models used. Lahmiri (2017) conducted research to provide a simple and practical method for predicting past volatility in currency exchange rates. The method forecasts the volatility of the US/Canada and US/Euro currency rates by feeding a limited collection of technical indicators into ANN and GARCH-type models. In terms of MAE, MSE and Theil's inequality coefficient, the basic ANN method beat the classic GARCH and EGARCH with different distribution assumptions, as well as the hybrid GARCH and EGARCH with ANN.

De Khoo et al. (2024) conducted a study on forecasting the volatility of stock indices by enhancing GARCH-type models through a combined weighted (CW) volatility measure and weighted volatility indicators (WVI). The study aimed to introduce a CW volatility measure and WVI that integrate return-based and range-based volatility metrics, evaluate the CW measure using five stock indices, finding that it resulted in the lowest losses compared to 5-min realized volatility, and investigate the incorporation of the CW measure and WVI as exogenous variables in GARCH-

type models to improve forecasting performance. The findings revealed that the CW volatility measure outperformed other measures in accurately estimating the true volatility of stock returns. Additionally, the GARCH-CW-WVI and EGARCH-CW-WVI models, which incorporated the CW measure and WVI as exogenous variables, demonstrated a better in-sample fit compared to standard GARCH and EGARCH models. Moreover, these enhanced models also exhibited superior out-of-sample forecasting performance relative to the standard GARCH and EGARCH models.

The study by Ogunnusi et al. (2024) examined the volatility of the Naira/US dollar exchange rate using the EGARCH (1, 1) model from the GARCH family, with a focus on the generalised t, skewed student t, and skewed normal distributions. The study used the monthly data from the Central Bank of Nigeria's spans from January 2003 to April 2023. The results showed that the skewed student t distribution outperformed the other distributions, showing superior predictive capability as indicated by higher log-likelihood, lower AIC, and reduced BIC values. Additionally, the forecast evaluation suggests that the variance reverts to a long-term mean. The study concluded that the choice of distribution is critical for improving model performance, with the skewed student t distribution identified as the most effective due to its flexibility in adapting to market conditions. This finding underscores the importance of selecting the right distribution for more accurate forecasting of exchange rate volatility.

The study by Rastogi et al. (2024) employed bivariate GARCH models (BEKK-GARCH and DCC-GARCH) to investigate how the volatilities of gold, crude oil, and interest rates impact the exchange rate (US dollar to Indian rupee). The study employed the daily data from January 2000 to December 2022. The results showed that both models indicate significant conditional covariance between the variables, confirming market connectivity theories. However, the DCC-GARCH model outperformed the BEKK-GARCH model in capturing the volatility spillover effects and conditional correlations, providing a more significant understanding of the relationships between the markets. Based on these findings, the study recommends that gold, crude oil, and interest rate markets be treated separately from exchange rate markets in risk management, with policymakers considering the resilience of these macroeconomic variables against exchange rate volatilities.

The study conducted by Charef (2024) utilised ANN to forecast financial time series and exchange rates in Tunisia, specifically the USD/TND, EUR/TND, and YEN/TND, using daily data from 2015 to 2019. The data was sourced from the Central Bank of Tunisia. The study applied two models: The Neural Network-GARCH (NN-GARCH) and the Multi-layer Perceptron-GARCH (MLP-GARCH) models, with the aim of enhancing the prediction of conditional variance. The findings showed that the hybrid NN-GARCH was the best fit model for exchange rates because it takes advantage of fundamental models inspired by artificial intelligence and alleviates their shortcomings model. The models' performance was compared using the Root Mean Square Error (RMSE) metric.

3. METHODOLOGY

The paper utilised publicly available secondary data, including exchange rates of the South African Rand compared to the Brazilian Real and Chinese Yuan, with each currency pair analyzed separately. A reason for selecting Brazilian Real and Chinese Yuan against South African Rand is the important role that Brazil and China play as major trade and investment partners for South Africa, particularly within the BRICS alliance and their strong economic connections. Analysing the volatility of these exchange rates can offer crucial insights into financial integration, trade dynamics, and the influence of macroeconomic policies across these emerging economies. The dataset covers the period from January 1996 to February 2024, consisting of 338 data points, with the exchange rate serving as the sole variable. The data was obtained from the website of the South African Reserve Bank (SARB) and analysed using Python. The data can be accessed through the following link: <https://www.resbank.co.za/en/home/what-we-do/statistics/releases/online-statistical-query>. The data does not have any missing values.

3.1. Stationarity

Exchange rates are volatile in nature and are among the many financial time series that are not stationary (Metsileng et al., 2021). Non-stationary data analysis raises new statistical issues. Unit root tests are used to determine whether non-stationarity exists and of what kind. Further information on these tests can be found in Hamilton (1994), Fuller (2009), Enders and Lee (2004), and Verbeek (2017). This article employed two unit root test procedures: the Augmented Dickey-Fuller (ADF) test and the Phillips-Perron (PP) test, which is an extension of the Dickey-Fuller (DF) test. The unit root test approaches are covered in the appropriate subsections that follow.

3.1.1. Augmented Dickey-Fuller (ADF) test

The original DF test was developed into the ADF test due to the low probability that the error term is white noise. By including extra lagged components for the dependent variables, Dickey and Fuller extended the test in order to address the autocorrelation problem and achieve stationarity. To put it simply, the current model is supplemented with the dependent variable's lag values. Until autocorrelation is abolished, this process is continued (Mushtaq, 2011). The ADF hypothesis is given as:

$$\begin{aligned} H_0: & \text{Unit root is present or series is non-stationary} \\ H_1: & \text{Unit root is absent or the series is stationary} \end{aligned}$$

The decision rule relies on the P-value generated by the test; if the P-value is below the significance level of 0.05, the null hypothesis is rejected in favor of the alternative hypothesis. This indicates that the series is stationary.

3.1.2. Phillips-Perron (PP) test

The Phillips-Perron (PP) unit root tests have become widely used in the analysis of financial time series. The main difference between the PP tests and the ADF tests lies in their approach to handling serial correlation and heteroskedasticity in the error terms. The PP test ignores serial correlation in the test regression, while the ADF tests use parametric autoregression to model the

ARMA structure of the errors (Phillips and Perron, 1988). The hypothesis of the PP test is given as:

H_0 : The time series has a unit root, reflecting non-stationarity

H_1 : The time series has no unit root, which makes it stationary

The time series is considered stationary, and the null hypothesis is rejected if the P-value associated with the PP test is below a predefined significance level, typically 0.05. Conversely, if the P-value is higher than the significance level, it indicates the presence of a unit root, suggesting that the time series is non-stationary, and the null hypothesis cannot be rejected. Therefore, differencing will be applied to make the series stationary.

3.2. Normality Tests

The tests that will be performed to determine whether the exchange rate data has a normal distribution are described in this subsection. The Jarque-Bera test for normality and residual normality plots were utilised for this step of the normality test.

3.2.1. Residual normality plots (QQ-plot and histogram distribution plot)

According to Loy et al. (2016), “based on visual inspection of a Q-Q plot, a sample is considered to be consistent with a normal distribution if the empirical and theoretical quantiles fall close to the line representing the theoretical distribution (acute line)”, while according to the Dietary Assessment Primer “a variable that is normally distributed has a histogram (or “density function”) that is bell-shaped, with only one peak, and is symmetric around the mean.” Figure 1 is a typical example of how a q-q and histogram plot would look like according to Social Science Computing Cooperative (SSCC) (2023).

3.2.2. Jarque-Bera (JB) test

The JB test is defined by Thadewald and Buning (2007) as “a goodness-of-fit assessment that gauges how closely sample data resembles a normal distribution in terms of skewness and kurtosis.” The JB test is expressed as follows:

$$JB = \frac{n}{6} * \left(S^2 + \frac{(k-3)^2}{4} \right) \quad (1)$$

where n is the sample size, S is the sample skewness, and k is the sample kurtosis. JB test hypothesis is as follows:

H_0 : The data is sampled from a normal distribution

H_1 : The data is not sampled from a normal distribution

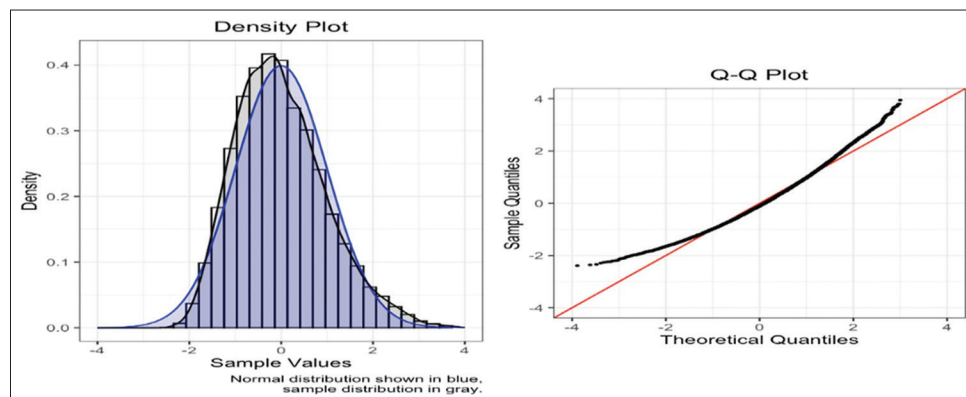
According to Tsay (2005), if the JB test P-value is less than the significance level (0.05), the null hypothesis that the data is sampled from a normal distribution is rejected in favour of the alternative hypothesis, indicating that the data is not sampled from a normal distribution. However, if the p-value exceeds the significance level, the null hypothesis cannot be rejected. The following section details the methods and models chosen for estimation in the current study.

3.3. Estimation Methods

This section presents the procedure that is followed in incorporating the ANN model into GARCH-type models, to create the GARCH-ANN hybrid models (GARCH-ANN, EGARCH-ANN, and GJR-GARCH-ANN). This paper adopted a similar methodological framework to that of Jannah et al. (2021) in the development of hybrid models. The procedure begins by estimating the initial GARCH model and extracting its residuals. These residuals are then used as input data for training the ANN component. Once the ANN is trained, it generates a series of predictions, which are subsequently integrated into the GARCH model.

The final step involves re-estimating the GARCH model using these ANN-generated outputs, resulting in a refined hybrid specification. Using GARCH residuals rather than raw returns as inputs to the ANN helps ensure that the neural network focuses specifically on capturing the nonlinear dynamics that the parametric model fails to explain (Zhang et al., 1998; Khashei and Bijari, 2011). This strategy reduces information overlaps between model components and enhances the interpretability of the hybrid system. In this structure, the ANN acts as a nonlinear adjustment layer, complementing the GARCH model and improving overall forecasting performance, particularly in scenarios involving extreme volatility or structural shifts that are not well handled by traditional GARCH models (García and Kristjanpoller, 2019.; McAleer, 2005). According to Mademlis and Dritsakis (2021), function y_1 is used to represent the output layer of ANN, otherwise called ANN predictions, while ξ_1 represents the coefficient for the ANN component in the asymmetric models. To the current study let $f(ANN \text{ predictions})$ represent y_1 .

Figure 1: Residual normality plots sourced from SSCC (2023)



3.3.1. GARCH-type models

The following are the GARCH-type models of interest: The standard GARCH; EGARCH and GJR-GARCH. The GARCH model was introduced by Bollerslev (1986) as an extension of the ARCH model. The GARCH model can capture volatility in the simplest form. The EGARCH model was developed by Nelson (1991). This GARCH extension was created as a response to the asymmetrical shortfall of the GARCH model of imposing non-negative constraints, in this sense Nelson introduced an extra function called the leverage effect, which according to Black (cited by Caporin and Costola, 2019). The GJR-GARCH model is a model with flexibility that takes conditional heteroskedasticity, or volatility clustering into account in the process of innovation. The general form of this model was introduced by Glosten et al., 1993 as an extension to the GARCH model. The GARCH (p, q) process is given by:

$$\sigma_t^2 = \alpha_0 + \sum_{i=1}^q \alpha_i \varepsilon_{t-i}^2 + \sum_{j=1}^p \beta_j \sigma_{t-j}^2 \quad (2)$$

If one let $p=q=1$ from GARCH (p, q) then the following are GARCH (1,1)-type model equations respectively:

$$\sigma_t^2 = \omega + \alpha_1 \varepsilon_{t-1}^2 + \beta_1 \sigma_{t-1}^2 \quad (3)$$

where σ_t^2 is the conditional variance, ω the constant term, β GARCH effect, and α ARCH effect (symmetric effect), and with only a few parameters (three to be specific) in the conditional variance equation is adequate to obtain a good model fit for exchange rate returns.

$$\ln(\sigma_t^2) = \omega + \beta_1 \ln(\sigma_{t-1}^2) + \alpha_1 \left\{ \left| \frac{\varepsilon_{t-1}}{\sigma_{t-1}} \right| - \sqrt{\frac{2}{\pi}} \right\} - \gamma \frac{\varepsilon_{t-1}}{\sigma_{t-1}} \quad (4)$$

Where ω is the constant term, $\beta_1 \ln(\sigma_{t-1}^2)$ denotes the fitted variance from the previous period, γ is the value of the leverage term if the value of $\gamma > 0$ then it is concluded that there is a larger impact for negative shocks on the conditional variance given as:

$$\zeta_t^2 = \omega + \sum_{i=1}^p \alpha_i \varepsilon_t^2 + \sum_{j=1}^q \beta_j \sigma_{t-j}^2 + \sum_{i=1}^p \gamma_i I_{t-i} \varepsilon_{t-i}^2 \quad (5)$$

Where $I_{t-i} = 1$ if $\varepsilon_{t-i} < 0$, or 0 if $\varepsilon_{t-i} \geq 0$.

3.3.2. ANN model

According to Thorat et al. (2022), ANN is a data-processing paradigm that is made up of numerous layers of simple processing components known as neurons. The neuron has two functions: Collecting inputs and generating outputs. The mathematical model of a network sheds insight into the concepts of inputs, weights, summation function, activation function, and output (Dongare et al., 2012). ANN works based on two principles which are forward propagation and backward propagation. Forward propagation, according to Luhaniwal (2019), is the act of feeding input data into a network in a forward manner to produce an output. Each hidden and output layer node of a neural network undergoes pre-activation and activation in this process; the pre-activation function is computed as a weighted sum, and the activation function is applied using bias to guarantee non-linear flow (Luhaniwal, 2019). The use of the chain rule will be applied for this subsection (backpropagation). The chain rule

demonstrates a method for calculating the derivative of a set of composite functions, where the number of functions in the composition determines the number of differentiation steps that must be taken (MathsCentre, 2009), for example, if a composite function $f(x)$ is defined as:

$$f(x) = (g \circ h)(x) = g[h(x)] \quad (6)$$

Then

$$f'(x) = g' [h(x)] * h'(x) \quad (7)$$

where g and h represent different functions or layers of the network and $h(x)$ represent the output of a hidden layer. Equation 7 is the derivative of equation 6. The primary objective of neural networks is to minimise error; to make this possible, all weights must be updated via backpropagation (Kostadinov, 2019). According to Mademlis and Dritsakis (2021), the backpropagation NN is derived by a weighted linear summation of the inputs in the following way:

$$\alpha_j = \sum_{i=1}^d w_{ij} x_i \quad (8)$$

To activate the hidden unit j , the linear summation of equation 6 by employing a logistic activation function $g(a)$ is transformed:

$$h_j = g \left(\sum_{i=1}^d w_{ij} x_i \right) \quad (9)$$

where

$$g(a) = \frac{\exp(a)}{(1 + \exp(a))} \quad (10)$$

then the neuron of the output layer is given by:

$$y_1 = g \left(\sum_{j=1}^3 w_{1j} g \left(\sum_{i=1}^d w_{ij} x_i \right) \right) \quad (11)$$

Where x_i corresponds to the number of inputs ($i = 1, 2, \dots, d$), j corresponds to the number of hidden neurons which are three ($j = 1, 2, 3$), w_{ij} are the weights from the input layer to the hidden layer and are the weights from the hidden layer towards the output layer.

3.3.3. The GARCH-ANN hybrid model

The GARCH-ANN model is the first hybrid model used in the current paper. The GARCH model was developed by Bollerslev (1986) to capture volatility as stated in the introduction section. Making use of the function $f(ANN \text{ predictions})$ which represents the ANN output layer (ANN predictions) as the incorporation of ANN into the GARCH model results into a possible formulation of the variance equation of the hybrid model expressed as:

$$\sigma_t^2 = \omega + \alpha_1 \varepsilon_{t-1}^2 + \beta_1 \sigma_{t-1}^2 + f(ANN \text{ predictions}) \quad (12)$$

In equation (12), σ_t^2 is the conditional variance, ω the constant term, β GARCH effect, α ARCH effect (symmetric effect), and $f(ANN \text{ predictions})$ represents the output layer of the ANN model.

3.3.4. The EGARCH-ANN hybrid model

The EGARCH-ANN model is the second used model in the article. This model is a combination of ANN and the EGARCH model,

introduced by Nelson (1991), as already mentioned addresses some limitations of the standard GARCH model, particularly by allowing for asymmetries in the volatility clustering process. When combining EGARCH with ANN, the aim is to leverage the nonlinear modelling capabilities of ANNs to further enhance volatility prediction. Combining the original EGARCH model and the ANN output layer component a possible formulation of the full variance of the hybrid model can be expressed as:

$$\ln(\sigma_t^2) = \omega + \beta_1 \ln(\sigma_{t-1}^2) + \alpha_1 \left\{ \left| \frac{\varepsilon_{t-1}}{\sigma_{t-1}} \right| - \sqrt{\frac{2}{\pi}} \right\} - \gamma \frac{\varepsilon_{t-1}}{\sigma_{t-1}} + \xi_1 f(ANN \text{ predictions}) \quad (13)$$

where ω is the constant term, $\beta_1 \ln(\sigma_{t-1}^2)$ denotes the fitted variance from the previous period, γ is the value of the leverage term, and $f(ANN \text{ predictions})$ represents the output of the ANN model, and ξ_1 is the coefficient for the ANN component.

3.3.5. The GJR-GARCH-ANN hybrid model

Finally, the third model, GJR-GARCH-ANN is a combination of ANN and GJR-GARCH models. The GJR-GARCH model is individually known as a model with flexibility that takes conditional heteroskedasticity, or volatility clustering, into account in the process of innovation, however in the hybrid model, the ANN component is used to capture nonlinearities and generate additional variables which might improve the GJR-GARCH model's accuracy. Thereby combining GJR-GARCH with the ANN model, a possible formulation of the hybrid model can be expressed as follows:

$$\zeta_t^2 = \omega + \alpha_1 \varepsilon_t^2 + \beta_1 \sigma_{t-1}^2 + \gamma_1 I_{t-1} \varepsilon_{t-1}^2 + \xi_1 f(ANN \text{ predictions}) \quad (14)$$

3.4. Model Evaluation Measures

The following section outlines the model evaluation measures employed to assess the performance and effectiveness of the models under consideration. The evaluation measures used in the paper are Mean Absolute Percentage Error (MAPE), Mean Square Error (MSE) and Mean Absolute Error (MAE).

3.4.1. Mean absolute percentage error (MAPE)

The MAPE, which according to Sidqi and Sumitra (2019) evaluates a forecasting system's accuracy. It can be calculated as the average absolute percentage error for each period between the predicted values and the actual values to represent this accuracy as a percentage, which according to Khair et al. (2017) can be presented as:

$$MAPE = \frac{1}{n} \times \sum_{t=1}^n \left| \frac{A_t - F_t}{A_t} \right| * 100 \quad (15)$$

Where n is the sample size, A_t is the actual value, F_t is the forecast value, and \sum is summation notation (the absolute value is summed for every forecasted point in time).

3.4.2. Mean square error (MSE)

MSE is a measure of how inaccurate statistical models are (Frost, 2023). It assesses the average squared difference between the

observed and predicted values. When a model is error-free, the $MSE = 0$. Its value increases when model error does as well. The MSE is computed by:

$$MSE = \frac{\sum (y_i - \hat{y}_i)^2}{n} \quad (16)$$

where y_i is the observed value, \hat{y}_i is the predicted value, and n is the sample size or number of data observations.

3.4.3. Mean absolute error (MAE)

The MAE is a measurement of the average size of the errors in a set of forecasts, without considering their direction (Anapedia, 2023). MAE is computed using the following equation (Wang and Lu, 2018):

$$MAE = \frac{\sum_{n=1}^n |\hat{r}_n - r_n|}{n} \quad (17)$$

where \hat{r}_n is the predicted value, r_n is the true value or observed values, and n is the number of observations in the dataset.

4. RESULTS AND DISCUSSIONS

This section of the article presents the results of the data analysis, and the discussion of the results obtained from the data analysis. Table 1 presents the results of the descriptive statistics. Descriptive statistics are used to describe the dataset used in the study.

According to Table 1, the ZAR/REAL currency pair has a mean value of 3.75 with a standard deviation of 0.71, while the currency pair of ZAR/YUAN produced a mean value of 1.41 with a standard deviation of 0.65, and of the two currency pairs, it can be noted that the ZAR/REAL pair produced the highest mean value. The ZAR/REAL currency pair reflected a skewness value of -0.50 which indicates that the left tail of the distribution is longer or fatter than the right tail (in other words, the ZAR/REAL distribution is negatively skewed), meanwhile, the ZAR/YUAN currency pair appears to be positively skewed with a skewness value of 0.42 , which indicates that the right tail of the distribution is longer or fatter than the left tail. Both currency pairs produced negative kurtosis values, with ZAR/REAL producing -0.47 and ZAR/YUAN with -1.21 , indicating distributions that are slightly platykurtic, meaning that both currency pairs have thinner tails and are less peaked than a normal distribution. Figure 2 presents the graphical presentation of the dataset.

Table 1: Descriptive statistics of ZAR/REAL and ZAR/YUAN

Variable	ZAR/REAL	ZAR/YUAN
Mean	3.75	1.41
Median	3.83	1.18
Maximum	5.40	2.73
Minimum	2.02	0.44
Standard deviation	0.71	0.65
Skewness	-0.50	0.42
Kurtosis	-0.47	-1.21

Table 2: ADF and PP tests of original data

Currency pairs	Level of test	ADF test statistic	P-value	PP test statistic	P-value
ZAR/REAL	Level	-2.4959	0.1165	-2.3367	0.1604
	1 st difference	-15.4469	<2.8174e-28***	-15.4416	<2.8606e-28***
ZAR/YUAN	Level	-0.4085	0.9087	-0.3415	0.9194
	1 st difference	-8.9875	<7.0371e-15***	-16.6875	<1.4981e-29***

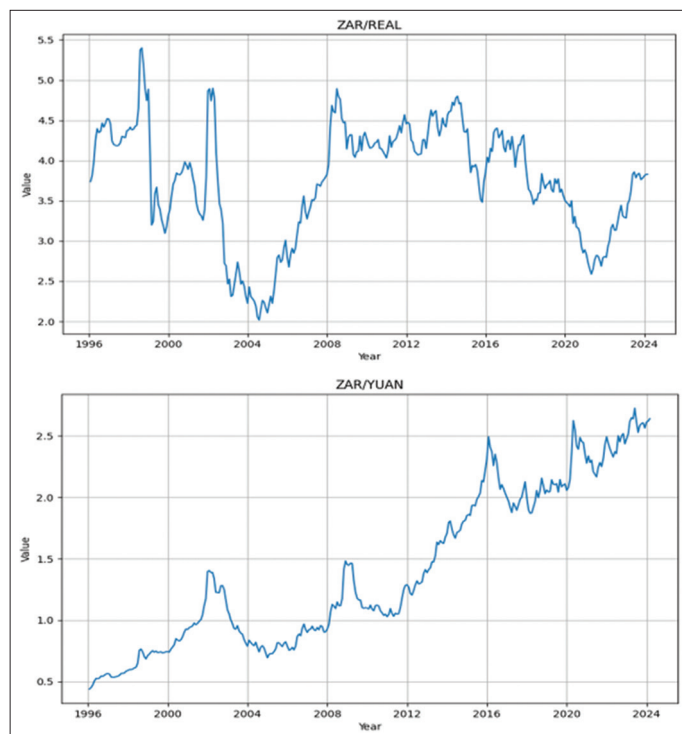
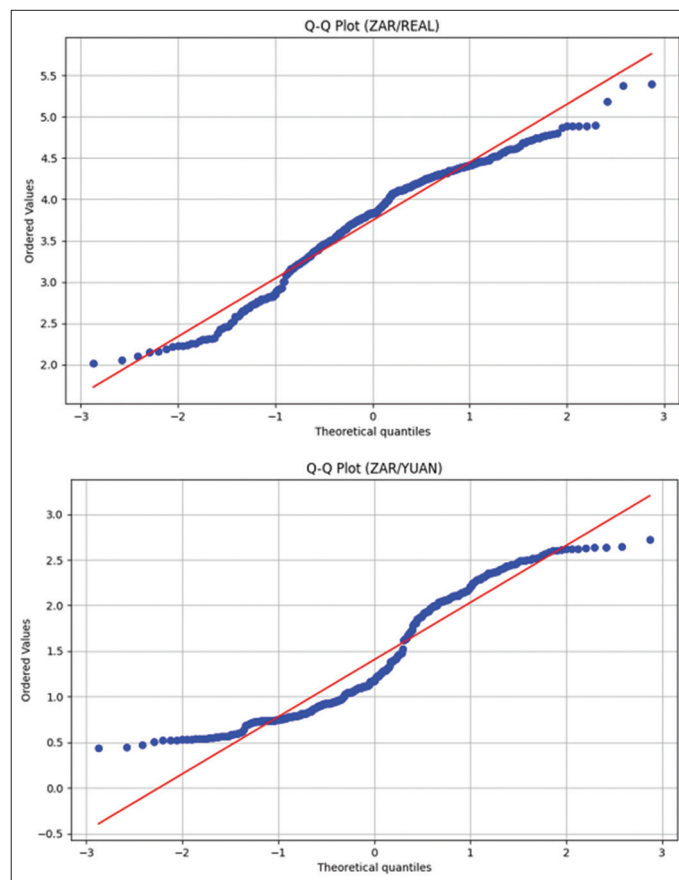
Figure 2: Original plots of the currency pairs

Figure 2 shows the two exchange rate pairs' original plots. The ZAR/REAL exchange rate plot shows a very unstable fluctuation through the entire time frame, with a huge drop between the period of 2003 and the end of 2004 which essentially meant a Real was less expensive compared to the period of 2005 to early 2009 which saw a sudden increase. It is also evident from that figure that there is a continuous fluctuation of small differences from 2010 to 2023. The ZAR/YUAN exchange rate plot reflects an upward trend in the long run (a constant increase with the Chinese Yuan getting stronger than the Rand with time), with a huge increase in the price of Yuan from 2005 to 2016, in entirety, there has been minor notable drops during certain years which usually lasted for periods not longer than a year, overall, a huge increase in the Chinese Yuan cost per South African Rand (ZAR). By eye inspection, it is concluded that the variables seem to be nonstationary and not normally distributed. The formal test for stationarity and normality was then computed and the results are presented in Table 2.

Table 2 presents the summary of the results for the ADF and PP tests. The results revealed that both currency pairs are nonstationary at level. However, both pairs became stationary at first difference. Figures 3 and 4 present the Q-Q and the histogram plots (Normality Residual Plots) for both the currency pairs respectively.

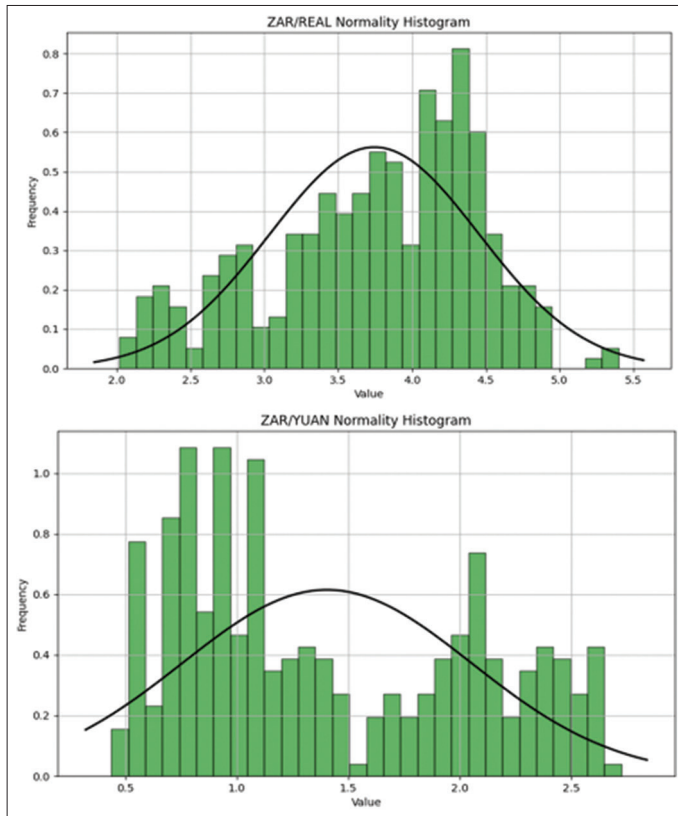
From Figure 3, it is visible from both currency pairs' plots that most of the exchange rate values fall close to the line representing the

Figure 3: Q-Q plots

theoretical distribution (acute line), therefore it can be concluded that from visual representation both currency pairs' series are normally distributed after differencing the data using power-transformation without outliers. Figure 4 presents histograms for both currency pairs and from visual inspection it can be confirmed that both the plots display the normality bell shape and the currency pairs' histograms have one peak, which then can be said that from visual inspection the currency pairs are normally distributed.

A more formal assessment was performed using the Jarque-Bera (JB) test to validate or refute the normality assumptions derived from the visual inspections of Figures 3 and 4. The results of this test are presented in Table 3.

The JB test results shown in Table 3 align with the visual representations in Figures 3 and 4. Both currency pairs yielded $P > 0.05$, which, according to the JB test decision rule, indicates that the null hypothesis that the data is sampled from a normal distribution cannot be rejected. This suggests that the data for both currency pairs is normally distributed. Tables 4-6 present the parameter estimation results for the three models used in

Figure 4: Histogram plots

the paper. Table 4 follows with the GARCH (1,1)-ANN model results.

From Table 4, the following models may be inferred. The GARCH (1,1)-ANN model equations for every currency pair are expressed as follows:

$$\text{ZAR / REAL} : \sigma_t^2 = 1.0007 + 0\epsilon_{t-1}^2 + 0.9314\sigma_{t-1}^2 + f(\text{ANN predictions}) \quad (18)$$

$$\text{ZAR / YUAN} : \sigma_t^2 = 0.0104 + 0\epsilon_{t-1}^2 + 0.9963\sigma_{t-1}^2 + f(\text{ANN predictions}) \quad (19)$$

The sums of the estimates α_1 and β_1 for both exchange rate pairs/currency pairs are <1 , which means that the unconditional volatility for both exchange rate pairs is finite. The results further revealed that both the currency pairs produced high volatility persistence values with ZAR/REAL: $\alpha_1 + \beta_1 = 0.9314$, while ZAR/YUAN produced. $\alpha_1 + \beta_1 = 0.9963$. Table 5 presents the EGARCH (1,1)-ANN parameter estimates.

Table 5 shows that the leverage effects, γ_i , of both the currency pairs are >0 (positive coefficients), which implies that an increase in the exchange rate has a greater impact on conditional volatility as compared to a decrease in the exchange rate. The impact for the two currency pairs appears to be strong (ZAR/REAL γ [0.0817] and ZAR/YUAN γ [1.1875]) since they are greater than the respective symmetric effects (ZAR/REAL α (-0.3974) and ZAR/YUAN α (-1.2713)). The relative size of the two groups of coefficients (γ and α) suggests that the asymmetric effects

Table 3: JB test results

Currency pairs	Level of test	JB test statistic	P-value
ZAR/REAL	1 st difference	0.3760	0.8290
ZAR/YUAN	1 st difference	0.0700	0.9660

Table 4: Summary results of GARCH (1,1)-ANN model parameter estimates

Currency pairs	Parameter	Estimate	Standard error	t-value	P-value
ZAR/REAL	ω	1.0007	1.1110	0.9010	0.3680
	α_1	0.0000	0.0116	0.0000	1.0000
	β_1	0.9314	0.0720	12.942	$<0.0000^{***}$
ZAR/YUAN	ω	0.0104	0.0364	0.2860	0.7750
	α_1	0.0000	0.0079	0.0000	1.0000
	β_1	0.9963	0.0154	64.664	$<0.0000^{***}$

***, **, and * indicates significant codes at 0.01, 0.05, and 0.1 respectively

Table 5: EGARCH (1,1)-ANN parameter estimates summary table

Currency pairs	Parameter	Estimate	Standard error	t-value	P-value
ZAR/REAL	ω	2.3115	1.0290	2.2460	0.0247**
	α_1	-0.3974	0.2540	-1.5660	0.1170
	β_1	0.1360	0.4300	0.3160	0.7520
	γ_i	0.0817	0.1990	0.4100	0.6820
ZAR/YUAN	ω	-0.0321	0.3450	-0.0929	0.9260
	α_1	-1.2713	0.5230	-2.4310	0.0151**
	β_1	0.0000	0.8140	0.0000	1.0000
	γ_i	1.1875	0.4970	2.3900	0.0168**

***, **, and * indicates significant codes at 0.01, 0.05, and 0.1 respectively

Table 6: Summary results of the GJR-GARCH (1,1)-ANN parameter estimates

Currency pairs	Parameter	Estimate	Standard error	t-value	P-value
ZAR/REAL	ω	1.8120	2.2140	0.8180	0.4130
	α_1	0.0000	0.0244	0.0000	1.0000
	β_1	0.8757	0.170	5.149	$<0.0000^{***}$
	γ_i	0.2117	0.292	0.724	0.4690
ZAR/YUAN	ω	0.0104	0.0419	0.249	0.8030
	α_1	0.0000	0.0069	0.000	1.0000
	β_1	0.9963	0.0168	59.240	$<0.0000^{***}$
	γ_i	0.0000	0.275	0.000	1.0000

***, **, and * indicates significant codes at 0.01, 0.05, and 0.1 respectively

dominate the symmetric. The currency pairs' stationarity is also assured by the past volatility coefficient β since they are <1 for both currency pairs, it is also notable that the β value implies that there is the presence of low shock persistence in the two currency pairs. From Table 5, the following models may be inferred. The EGARCH (1,1)-ANN model equations for every currency pair are expressed as follows:

$$\begin{aligned} \text{ZAR / REAL} : \ln(\sigma_t^2) = & 2.3115 + 0.1360 \ln(\sigma_{t-1}^2) \\ & - 0.3974 \left\{ \frac{\epsilon_{t-1}}{\sigma_{t-1}} - \sqrt{\frac{2}{\pi}} \right\} - 0.0817 \frac{\epsilon_{t-1}}{\sigma_{t-1}} + \xi_1 f(\text{ANN predictions}) \end{aligned} \quad (20)$$

Table 7: Model evaluation measures results

MODELS\CURRENCY	ZAR/REAL			ZAR/YUAN		
	MSE	MAE	MAPE (%)	MSE	MAE	MAPE (%)
GARCH (1,1)-ANN	0.0740	0.2141	6.34	0.0107	0.0841	7.54
EGARCH (1,1)-ANN	0.0665	0.2031	6.02	0.0082	0.0731	6.30
GJR-GARCH (1,1)-ANN	0.0740	0.2149	6.35	0.0107	0.0841	7.54

Table 8: Summary results of the models' diagnostic tests

MODELS\CURRENCY	ZAR/REAL		ZAR/YUAN	
	P-values		P-values	
	ARCH-LM	LJUNG-BOX	ARCH-LM	LJUNG-BOX
EGARCH (1,1)-ANN	0.3875	0.4474	0.8806	0.9212

***, **, and * indicates significant codes at 0.01, 0.05, and 0.1 respectively

$$\begin{aligned} \text{ZAR / YUAN : } \ln(\sigma_t^2) &= -0.0321 + 0 \ln(\sigma_{t-1}^2) \\ &- 1.2713 \left\{ \left| \frac{\varepsilon_{t-1}}{\sigma_{t-1}} \right| - \sqrt{\frac{2}{\pi}} \right\} - 1.1875 \frac{\varepsilon_{t-1}}{\sigma_{t-1}} + \xi_1 f(\text{ANN predictions}) \end{aligned} \quad (21)$$

The sums of the estimates α_i and β_i of both the currency pairs are <1 , this means that the unconditional volatility of the two currency pairs is finite. The output further showed that currency pairs have low volatility persistence values with ZAR/REAL: $\alpha_i + \beta_i = -0.2614$ and ZAR/YUAN: $\alpha_i + \beta_i = -0.2713$. Table 6 follows with the parameter estimates results of the GJR-GARCH (1,1)-ANN model.

Table 6 shows that the leverage effects, Y_j , of both the currency pairs are ≥ 0 (positive coefficients), which implies that an increase in the exchange rate has a greater impact on conditional volatility as compared to a decrease in the exchange rate. The impact for the two currency pairs appears to be strong (ZAR/REAL Y [0.2117] and ZAR/YUAN Y [0]) since they are greater or equal to the respective symmetric effects (ZAR/REAL (0) and ZAR/YUAN (0)). The relative size of the two groups of coefficients (Y and α) suggests that the asymmetric effects dominate the symmetric for the ZAR/REAL currency pair, while the ZAR/YUAN currency pair is equally balanced. The currency pairs' stationarity is also assured by the past volatility coefficient β since they are <1 for both currency pairs, it is also notable that the β value implies that there is the presence of high shock persistence in the two currency pairs. From Table 6, the following models may be inferred. The GJR-GARCH (1,1)-ANN model equations for every currency pair are expressed as follows:

$$\begin{aligned} \text{ZAR / REAL : } \zeta_t^2 &= 1.8120 + 0\varepsilon_t^2 + 0.8757\sigma_{t-1}^2 + 0.2117I_{t-1}\varepsilon_{t-1}^2 \\ &+ \xi_1 f(\text{ANN predictions}) \end{aligned} \quad (22)$$

$$\begin{aligned} \text{ZAR / YUAN : } \zeta_t^2 &= 0.0104 + 0\varepsilon_t^2 + 0.9963\sigma_{t-1}^2 + 0I_{t-1}\varepsilon_{t-1}^2 \\ &+ \xi_1 f(\text{ANN predictions}) \end{aligned} \quad (23)$$

The sums of the estimates α_i and β_i of both the exchange rate pairs are <1 . This means that the unconditional volatility of all exchange rate pairs is finite. The results further revealed that the two currency pairs produced high volatility persistence, with ZAR/REAL $\alpha_i + \beta_i = 0.8757$ and ZAR/YUAN $\alpha_i + \beta_i = 0.9963$. Table 7 presents a summary of the models' evaluation measures.

The outcomes obtained in Table 7 also revealed that the hybrid EGARCH (1,1)-ANN model had the best overall performance when compared to the other hybrid models (GARCH (1,1)-ANN and GJR-GARCH (1,1)-ANN) based on all three evaluation measures, with the lowest MSE of 0.0665, MAE of 0.2031, and MAPE of 6.02% for the ZAR/REAL currency pair, while for the ZAR/YUAN currency pair, the same hybrid EGARCH (1,1)-ANN model performed better than all the other hybrid models based on all three evaluation measures with MSE of 0.0082, MAE of 0.0731, and MAPE of 6.30%. Table 8 summarises the diagnostic checks of the best performing hybrid model.

The results in Table 8 show that for both currency pairs, the EGARCH (1,1)-ANN model has no ARCH errors, since the P-values of the ARCH-LM test are >0.05 level of significance. This essentially means according to the ARCH-LM test hypothesis and decision rules, since the P-values are >0.05 , the null hypothesis (The series has no ARCH effect, or Residuals are homoscedastic) is therefore rejected in favor of the alternative hypothesis (The series has ARCH effect, or residuals are conditionally heteroskedastic). Therefore, according to the ARCH-LM test, the model can be used for possible forecasts under both currency pairs.

Table 8 further revealed that the EGARCH (1,1)-ANN model under both currency pairs (ZAR/REAL and ZAR/YUAN), the Ljung-Box test showed that the data appears to have serial correlation (not normally distributed) since the P-values are >0.05 . Overall, the EGARCH (1,1)-ANN model is deemed fit to be used for possible volatility forecasting if that need arises by the ARCH-LM test. However, the Ljung-Box test produced differing results which deemed the model not fit for future forecasts.

5. CONCLUSION

The ANN was incorporated into GARCH-type models to investigate the performance of improved GARCH-type models, and this incorporation gave birth to GARCH-ANN hybrid models, and the GARCH-ANN hybrid models: GARCH (1,1)-ANN, EGARCH (1,1)-ANN, and GJR-GARCH (1,1)-ANN, were fitted and the results were also presented. The results showed that the three hybrid models had acceptable performance based on the evaluation measures. The hybrid models produced MSE, MAE, and MAPE values which indicate good accuracy and at this stage

fitness of the models to be used for possible future forecasts pending diagnostics. Kristjanpoller and Minutolo (2015) supported the above assertion.

The results revealed that the EGARCH (1,1)-ANN model had the best overall performance when compared to the other hybrid models (GARCH (1,1)-ANN and GJR-GARCH (1,1)-ANN) based on all three evaluation measures, with the lowest values for the MSE, MAE, and MAPE for both the currency pairs' data (ZAR/REAL and ZAR/YUAN). The findings are supported by Hajizadeh et al. (2012). The hybrid models were fitted successfully, and the results showed that incorporating ANN into univariate GARCH models to create hybrid GARCH-ANN models can model exchange rate volatility of both currency pairs. The hybrid models were fitted successfully, and the results showed that incorporating ANN into univariate GARCH models to create hybrid GARCH-ANN models can successfully model exchange rate volatility of both currency pairs. The paper recommends further similar studies to predict future exchange rate trends. The current study compares hybrid models with traditional GARCH models. It is recommended that future studies may look at comparisons with other recent advanced forecasting techniques, such as support vector machines, gradient boosting, or other machine learning-based volatility models.

The study's findings highlight important economic implications, particularly for financial market participants, policymakers, and international trade. The superior performance of the EGARCH (1,1)-ANN model suggests that hybrid GARCH-ANN models can enhance exchange rate volatility forecasting, improving risk management for businesses and financial institutions. Policymakers can leverage these models to formulate more effective monetary and fiscal policies, while investors can use them to mitigate currency risk. The study also emphasises the value of integrating machine learning techniques into traditional econometric models, paving the way for more advanced forecasting methods in future research.

The use of monthly frequency data for the currency pairings, which could not adequately represent the high-frequency dynamics and intraday volatility patterns frequently seen in financial markets, is one of the study's main limitations. On shorter time periods, such as daily or hourly, exchange rate volatility might show notable fluctuations, which could be crucial in specific trading or risk management situations. The use of monthly data may limit the study's capacity to fully capture the breadth of market behaviours by ignoring the effects of short-term shocks and liquidity changes. Higher-frequency data, such as daily or intraday observations, may be used in future studies to better understand the short-term volatility structure and how it affects predicting accuracy. The scope of the study is limited to two currency pairs.

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