



A Stochastic Approach for Determining Profit Rate of Islamic Financing Products

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ABSTRACT

Islamic Bank in Malaysia uses Bai' Bithaman Ajil method and Musharakah Mutanaqisah Partnership method for the home financing process. However, these house price calculation methods are still similar to conventional computation. In addition, profit rate in the Islamic bank still refer interest rate as benchmark. The objective in this study is to determine a profit benchmark by using stochastic forecasting. Base financing rate data from the year 1999 until 2014 is used in this study to forecast a profit for home financing and to find an optimum price. The exponential brownian motion model with drift is one of the stochastic forecasting models which has been identified as the best model to forecast the data in this study. After that, the probability of profit is compared to choose the best profit. The median of a profit distribution is proposed as a benchmark for determining the house price. This propose benchmark is fair to both bank and customer in terms of profit and installment, respectively.

Keywords: Forecasting, Risk, Analysis

JEL Classifications: G170

1. INTRODUCTION

The study of Islamic banking is increasingly widespread in this era. The establishment of Bank Islam Malaysia Berhad in 1983 is a starting existence Islamic finance in Malaysia. According to Hassan (2011), one of the objectives of Islamic banking is free of *riba*, fraud and injustice. *Riba* is main issue in the Islamic and conventional banks. A primary concern in Islamic banking is avoiding *riba* in the pricing of goods or services. This matter is because of calculation of house financing in the Islamic bank is similar as the calculation of loan interest in conventional banking. Conventional loan, the profits are based on interest rates instead of a sales contract. For Islamic bank, earning profits through loan is a *riba*. *A riba* is prohibited in Islam. Tahir (2003) states that there are 15 issues were identified in Islamic banking. One of the issues is the pricing of risk management for Islamic financial products.

Pricing is one of the most important elements of marketing. Commonly, house price is compose of total installment plus profit. In the conventional bank involves base lending rate (BLR)

while in the Islamic Bank involves the base financing rate (BFR). Overnight policy rate is determined by Bank Negara Malaysia, is a benchmark for the BFR and BLR. BFR is a profit benchmark of product in the Islamic bank. Every Islamic bank has different BFR value. BFR value is fluctuating and should not forecast a point value only. According to Hakan and Gulumser (2011), when interest rates increase, deposits in conventional banks will increase and loan will decrease. In the Islamic bank the changes is interest rates will greatly affect to the deposits than loans. The Islamic banks use Murabahah method, Bai' Bithaman Ajil (BBA) method and the Musharakah Mutanaqisah Partnership (MMP) method in the selling process and loan process. Murabahah method is the concept of buying and selling. The BBA is a contract that applies Sharia-based for house financing while the MMP method is equity financing based contract. The BBA method determines a profit on a house at the beginning of the contract agreement. While for the MMP method, the rental rate based on the market rental rates. However BBA method and the MMP method still depend on the market interest rate. Therefore, this study will propose a profit benchmark by using a stochastic forecasting process. Furthermore,

this study aims to forecast the amount of the installment price of a house.

The study will be organized as follows. Part 2 will discuss literature review. In Part 3 will discuss the methodology of the study. Part 4 will discuss the results and discussion and the conclusion of the study in Part 5.

2. LITERATURE REVIEW

Since the 1960s, the study of price modelling has been conducted. The existing pricing models are the random walk hypothesis discounted cash flow model, capital asset pricing model (CAPM), reliability pricing model and binomial asset pricing model. CAPM is the most popular model from Sharpe (1964) and Lintner (1965).

Meera and Razak (2009) compared the BBA and MMP contracts. These contracts are commonly used in the Islamic banks for house financing. MMP contract is a combination of partnership (musharakah) and leases (Ijara). The BBA is an addition of the murabahah contract. The installment in Murabahah contract within a short period while installment in BBA contract over a long period. The profit rate in the BBA contract depends on market interest rates. BBA calculation method similar as the conventional calculation method. The difference between the conventional and BBA method is BBA method only has aqad purchase. Furthermore, the profit rate charged by the BBA contract is fixed during aqad. Mostly customer would change the BBA method to conventional method when the interest rates are low. The concept of MMP method is to use two separate contracts, which are partnership contract and hire contract. The MMP method uses market rental value to calculate the profit. MMP calculation method similar as calculating loan conventional method. The different in calculation method for MMP and conventional is the calculation of profit. The conventional method used the interest rate while the MMP method used rental rates. There is a suggestion for MMP method by using a house price index to set a profit benchmark. Rebate will given if customer pay lumpsum by using BBA method, but does not have an explanation a spesific rebate value in the contract. This study compares the interest rate and the average gross rented property in Kuala Lumpur in 1984 to 2005. In addition, this study also found that MMP method is better than the BBA method.

Razak and Taib (2009) found that most customers stated that the BBA similar as the conventional method. BBA profit rate uses BLR as a benchmark. BBA uses the annuity formula that is similar as the conventional method. In addition, the profit is fixed at the beginning of the contract. Otherwise, the MMP method is a fair method to be applied. Rental rates for MMP method based on the actual market value. Rental rate depends on the type and location of the house. Daud (2009) stated that the BBA is a deferred payment sale contract. This method determines a fix selling price and profit rate at the beginning of the contract. The profit rate is determined by market interest rates.

Rozita (2011) stated that the conventional bank used interest rate in a pricing model. The interest rate is the percentage of the basic charge for a certain period in the lending process. London

Interbank Offered Rate and BLR are common interest rates that are used in Malaysian conventional banks. Interest rates are always changing with time. The interest rate is said to be dependent random variables if they have a long memory which the past value affects the value in the future. An example of factors that influence interest rate is the financial policy of the government. Thus, the interest rate can be modelled to improve the profitability and forecast interest rate value.

Based on the previous studies, an Islamic bank still does not have a solution on the issue of asset pricing. An interest rate is used in determining the profit. BBA uses a current fixed rate during the declaration of aqad to buy the house. But characteristic of interest rate fluctuating from time to time. Therefore, this study proposes a way to determine profits by using a stochastic forecasting method.

3. METHODOLOGY

The BFR data from 1999 to 2014 is used in this study. Banks in Malaysia commonly use the BLR for the conventional bank and BFR for an Islamic bank. BFR is used to get a profit benchmark for an islamic bank products, for example profit benchmark for a house pricing. The BFR data need to be forecasted in order to obtain future profit for a house will know.

Forecasting is a part of time series analysis. There are two methods of forecasting which are qualitative forecasting and quantitative forecasting. This study focus on quantitative forecasting. There are three types of quantitative forecasting, which are time series forecasting, cross-sectional forecasting and mixed-panel forecasting. Historical data is involved in this study so that time series forecasting is used. There are several methods to forecast time series data which are Box-Jenkins ARIMA, classical decomposition method of time series, regression methods, non-linear extrapolation methods and stochastic methods. The stochastic forecasting method is used in this study because the interest rate data is volatile and random. This method is also suitable to use when the data has missing values.

The characteristic of the interest rate are changing with time. It is represented as a random variable and assume to be an independent variables. Due to these properties, the interest rate for a period does not affect to another period. Thus, the stochastic approach can give a good approximation.

Stochastic is a random process and independent of time. The basic steps for stochastic modelling are to determine the sample space, then determine the probability of a sample element. Next, any result set is determined and finally the probability of the results is calculated. Markov process is one of the stochastic process. There are two types of Markov process which are discrete time and continuous time. Discrete time Markov process is used in this study which are random walk brownian motion process, exponential random walk brownian motion process, mean reversion process and jump-diffusion process.

3.1. Random Walk Brownian Motion Process

If the process is modelled in the form of Brownian motion stochastic differential equation, the formula such as the Equation (1).

$$S_t = S_0 + \int_0^t \mu_s ds + \int_0^t \sigma_s dW_s \tag{1}$$

Where,

S_t = Asset price,

μ = Annualized growth or drift rate,

σ = Annualized volatility,

$W(t)$ = Standard Brownian motion.

In the differential equation is usually written in differential form as Equation (2).

$$dSt = \mu dt + \sigma dWt \tag{2}$$

The following is the characteristics of the random walk Brownian motion $\{B(t), t \geq 0\}$:

1. $B(0) = 0$
2. $\{B(t), t \geq 0\}$ is fixed and not dependent increase
3. $B(t)$ is a normal distribution with $E[B(t)] = 0$ and $Var[B(t)] = \sigma^2 t$.

Standard brownian motion is called if $\sigma = 1$ and $\{B(t), t \geq 0\}$. For a standard Brownian motion $\{W(t), t \geq 0\}$ with parameters σ such as equation (3).

$$B(t) = W(t) \tag{3}$$

Figure 1 is an example standard Brownian motion graph. The equation for the Brownian motion with drift as the Equation (4).

$$X(t) = \mu t + \sigma W(t) \tag{4}$$

Brownian motion with drift $\{X(t), t \geq 0\}$ in the stochastic process is called if it meets the following criteria:

1. $X(0) = 0$
2. $\{X(t), t \geq 0\}$ is fixed and not dependent increase
3. $X(t)$ has a normal distribution with a mean μt and variance $t\sigma^2$.

Figure 2 is an example of a random walk Brownian motion graph.

3.2. Exponential Brownian Motion Process

When $\{Y(t), t \geq 0\}$, it is the Brownian motion with drift process. Then, the process $\{X(t), t \geq 0\}$ is $X(t) = e^{Y(t)}$ called as geometric Brownian motion. Geometric Brownian motion also known as exponential Brownian motion. Therefore, the equation of this process such as Equation (5).

$$X(t) = e^{\mu t + \sigma W(t)} \tag{5}$$

Exponential random walk Brownian motion cannot be a negative value, while the random walk Brownian motion can be negative values. If the exponential random walk Brownian motion process is modelled in the form of the stochastic differential equation, the equation as Equation (6).

$$dSt = \mu St dt + \sigma St dWt \tag{6}$$

Figure 3 is an example exponential random walk Brownian motion with drift graph. The value of μ is the average absolute logarithmic

relative returns $\ln \left[\frac{S_t}{S_{t-1}} \right]$, while σ is the standard deviation for all values $\ln \left[\frac{S_t}{S_{t-1}} \right]$ are fixed to estimate the drift rate parameter and volatility parameter based on time series data set.

Figure 1: Standard Brownian motion graph

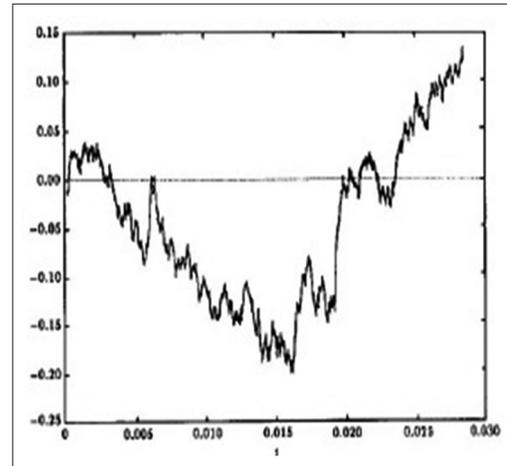


Figure 2: Random walk Brownian motion with drift graph

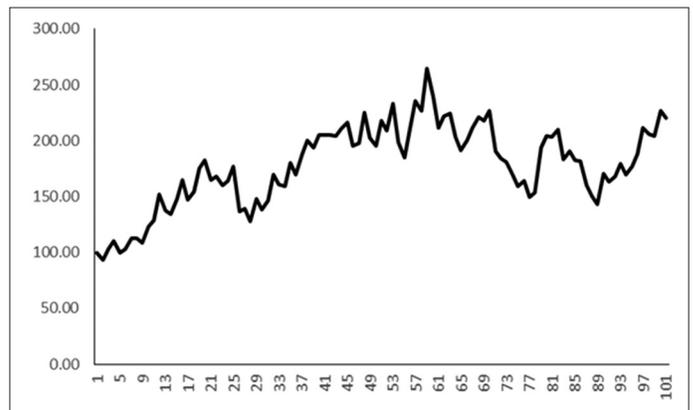
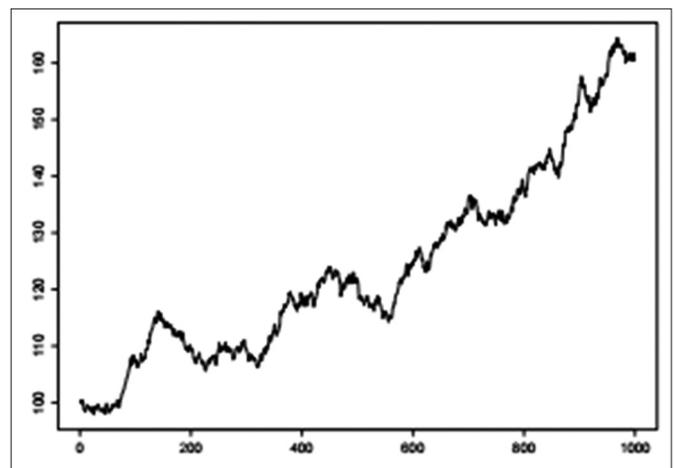


Figure 3: Exponential random walk Brownian motion with drift graph



3.3. Mean Reversion Process

The basis of the mean reversion process is Ornstein-Uhlenbeck arithmetic model. The difference between this process and Brownian motion process is a mean reversion process involves the nonstationary volatility. The difference between the mean reversion process and the random walk is the random walk will not return to the past mean level after the shocks occur. Meanwhile, the mean reversion is that market prices will return to the mean level before the shocks occur. The benefit of mean reversion process is that investors can forecast based on historical information. Figure 4 is an example mean reversion with drift process graph.

Equation (7) shows the mathematical structure for the mean reversion process.

$$dS_t = \eta(\mu - S_t) dt + \sigma dW_t \tag{7}$$

Equation (8) also can be used in this process.

$$\frac{\delta S}{S} = \eta(\bar{S}e^{\mu(\delta t)} - S)\delta t + \mu(\delta t) + \sigma\epsilon\sqrt{\delta t} \tag{8}$$

Where mean reversion parameter (η) and long term rate (S) obtained from historical data regression analysis.

$$Y_t - Y_{t-1} = \beta_0 + \beta_1 Y_{t-1} + \epsilon$$

$$\eta = -\ln [1 + \beta_1] \text{ and } \bar{S} = -\frac{\beta_0}{\beta_1}$$

Where,

η is mean reversion rate,

\bar{S} is long term value,

Y is data series,

β_0 is an intercept coefficient in regression analysis,

β_1 is a slope of the regression coefficients.

3.4. Jump Diffusion Processes

The process is same as the random motion but this process involves probability changes dramatically at any point of time. The equation for this process as Equation (9).

$$\frac{\delta S}{S} = \eta(\bar{S}e^{\mu(\delta t)} - S)\delta t + \mu(\delta t) + \sigma\epsilon\sqrt{\delta t} + \theta F(\lambda)(\delta t) \tag{9}$$

Where,

θ is the size of the jump S ,

$F(\lambda)$ is the inverse Poisson cumulative distribution,

λ is the jump rate S .

The process is the same as the random Brownian motion, but this process has a probability of jump point. The size of the jump is the ratio of the value post-jump to the pre-jump are calculate based on historical data. The jump diffusion process will involve two processes, which are Poisson process and Brownian motion process. The Poisson process involves frequency jumps while Brownian motion process involves the diffusion process. Figure 5 is an example of a jump diffusion process with drift graph.

Figure 4: Mean reversion with drift graph

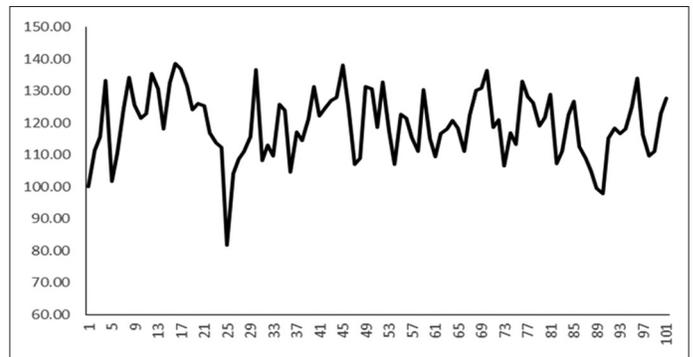
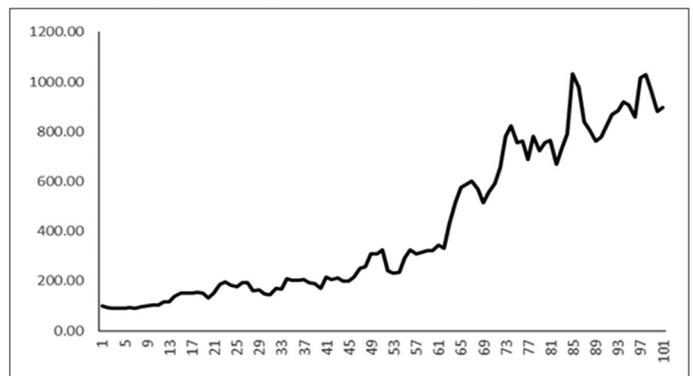


Figure 5: Jump diffusion with drift processes



3.5. Forecast Accuracy

There are several methods to measure forecast accuracy as the Equations (10-13). These methods will involve the forecast error, $e_t = y_t - f_t$ and n the number of data. This study will use the Equation (13) to calculate the forecast accuracy models because more sensitive in the scale and data changes. The accuracy values are smaller and more accurate in comparison with other methods. Where y_t original value and f_t forecast value.

Mean absolute error (MAE):

$$MAE = \frac{1}{n} \sum_{t=1}^n |e_t| \tag{10}$$

Mean absolute percentage error (MAPE):

$$MAPE = \frac{1}{n} \sum_{t=1}^n \left| \frac{e_t}{y_t} \right| \times 100 \tag{11}$$

Mean square error (MSE):

$$MSE = \frac{1}{n} \sum_{t=1}^n e_t^2 \tag{12}$$

Root MSE (RMSE):

$$RMSE = \sqrt{\frac{1}{n} \sum_{t=1}^n e_t^2} \tag{13}$$

3.6. Goodness of Fit Test

There are several test to decide the specific distribution which are Kolmogorov–Smirnov (KS) test, Chi-square test,

Anderson-Darling, Lilliefors, Jacquare-Bera, and Wilkes-Sharpipro. KS test is one of the most commonly used.

KS test based on the empirical distribution function. It only applies to continuous distributions. It is also to be more sensitive near the centre of the distribution. Given N ordered data points, so the empirical distribution function as Equation (14).

$$E_n = n_i/N \tag{14}$$

Where,
 n_i : The number of points less than Y_i where $Y_i = Y_1, Y_2, \dots, Y_N$

Equation KS statistic as Equation (15):

$$KS = \max_{1 \leq i \leq N} \left| F(Y_i) - \frac{i}{N} \right| \tag{15}$$

Where,
 F : The theoretical cumulative distribution of the continuous distribution.

4. RESULTS AND DISCUSSION

This section has five parts which are identification the best model, forecasting process, identification of profit distribution, simulation of profit and argument of 4% rate by using conventional method and BBA method. The first step to identify the best model is estimate the parameter by using BFR from 1999 to 2010. Then, the parameter is used to forecast BFR from 2010 to 2014 by using all models in the stochastic forecasting method. Next, the RMSE value of each model is compared to determine the best model. The best model will used to forecast 2015 onwards.

Before we begin estimate the parameters, simple statistical analysis was used to describe the data. Table 1 shows descriptve statistics results for BFR values from year 1999 to 2010 and from year 1999 to 2014.

This study uses yearly data. The sample size for BFR data from 1999 to 2010 is 11. The mean BFR is 2.86 and the standard deviation is 0.42. The median BFR of 2.71 indicates that 50% of the data have values lower than 2.71 and the remaining is more than 50%. Based on the Table 1, the value of mean is far from the median, so it can be expected that the data is not normally distributed.

The sample size for overall sample is 16. The mean BFR is 2.86 and the standard deviation is 0.37 and the median BFR is 2.81. Table 2 is a descriptive statistics table for BFR values from year 1999 to 2014.

4.1. The Process of Identifying the Best Model

First, the parameters such as drift rate, volatility, reversion rate, long term value, jump rate and jump size are estimated. Table 3 shows the parameter estimate in percentage for BFR from 1999 to 2010. The estimated parameters will be used to determine the best model and forecasting for year 2010 to 2014. The drift

rate and volatility are measured by the annual rate, while the reversion rate, the long-term value, the jump rate and jump size are measured by periodical data. Table 3 using the parameter estimate, Equation (16) is derived base on Equation (9).

$$\frac{\delta S}{S} = -0.2S_t dt + 0.2S_t dW_t + 1.6(2.83e^{-0.04(\delta t)} - S)\delta t + 0.06F(0.47)\delta t \tag{16}$$

This stochastic equation is used for forecasting BFR for the period 2010 to 2014. This equation consist of three components. First, $-0.2S_t dt + 0.2S_t dW$ is the exponent random walk brownian motion with drift. Second, $1.6(2.83e^{-0.04(\delta t)} - S)\delta t$ represents the mean reversion with drift. Third, $0.06F(0.47)\delta t$ is the jump diffusion with drift.

In this study, three models are considered for forecasting: Random walk Brownian motion with drift model, exponent random walk Brownian motion with drift model and jump diffusion with drift model. The mean reversion process model and mix process with drift was excluded because they have large errors. The reversion rate and long term parameter values were large, therefore the two models are not appropriate to forecast this data.

Then, the RMSE value of each model is compared to determine the best model that will be used to forecast. The best model must have small RMSE value. Table 4 shows that the exponent random brownian motion process with drift has the lowest RMSE values.

Table 1: Desriptive statistics from year 1999 to 2010 and from year 1999 to 2014

Description	Value (in sample, 1999-2010)	Value (overall, 1999-2014)
Mean	2.86	2.86
Median	2.71	2.81
Minimum	2.05	2.05
Maximum	3.44	3.44
Range	1.39	1.39
Standard deviation	0.42	0.37

Table 2: Value of parameter estimates from 1999 to 2014

Parameter	Value
Drift rate	-0.20%
Volatility	17.30%
Reversion rate	158.62%
Long term value	2.83
Jump rate	6.67%
Jump size	0.47

Table 3: Parameter estimates value from 1999 to 2010

Parameter	Value
Drift rate ¹	-4.26%
Volatility ²	19.45%
Reversion rate ³	264.92%
Long term value ⁴	2.83
Jump rate ⁵	10%
Jump size ⁶	0.54

¹ μ =Average $\ln[S_t/S_{t-1}]$, ² σ =Standard deviation all $\ln[S_t/S_{t-1}]$, ³ $\eta = -\ln[1+\beta]$, ⁴ $\bar{s} = -\beta/\beta$, ⁵rate from Poisson distribution, ⁶ $\frac{\text{post-jump value}}{\text{pre-jump value}}$

4.2. Forecasting Process (2015 onwards, 2015-2034)

Exponential random walk Brownian motion with drift model is chosen because it has the lowest RMSE compared to other models. By using data from 1999 to 2014, the parameter outsample is estimated again as shown in Table 2.

Using the parameter estimate, Equation (17) is derived base on Equation (9). This stochastic equation is used to forecast BFR for the period 2015 onwards.

$$\frac{\delta S}{S} = -0.2S_t dt + 0.2S_t dW_t + 1.6(2.83e^{-0.04(\delta t)} - S)\delta t + 0.47F(0.06)\delta t \tag{17}$$

Data for the next 20 years are forecasted by starting with the mean BFR rate in 2014 (3.07%), the drift rate (-0.2%) and the volatility (17.30%). The 20 years forecasting value 20 iterations are simulated. Figure 6 shows the 20 iterations which are forecasted stochastically.

Based on Table 5, the total installment for each iteration is calculated by using profit rate that has been forecasted. The table of amortization is given in the Appendix Tables 1-5. The value of the minimum, maximum, mean and standard deviation of BFR are obtained from 20 iterations in 20 years. The range of the total installment is between RM94,357.24 to RM138,327.73. The mean of total installments for 20 iterations is RM222,864.01. Profit is the difference of the total installment and principal amount¹. The principal amount is RM80,000. Therefore, minimum profit would be obtained base on the total installment is RM14,357.24 and maximum profit are RM58,327.73.

4.3. Identifying the Distribution of Total Profit (Total Installment-Initial Capital)

Next, the best distribution for total profit is determined. The purpose of distribution fitting is to calculate probability to achieve the profit and to simulate 1000 total profit by using the best distribution. The best distribution is determined by using the KS test.

Based on Table 6, parabolic distribution is the best to be used in this study with minimum RM2,670.70 and maximum RM60,128.60. From the simulation, the mean total installment is installment is RM111,864.11 and estimated profit range is RM14,357.24 to RM58,327.73.

4.4. Measure of Risk and Value at Risk

The minimum profit from Table 5 is RM14,357.24². There is only 10% chance to earn profit less than RM14,357.24. Most of the time (90%) chance to earn profit more than RM14,357.24. Therefore, if the bank sets profit less than RM14,357.24, the bank is offering low house loans but will lose a huge opportunity to earn profit more than RM14,357.24.

1 Profit = Total installment-RM80,000 (example: RM94,357.24-RM80,000 = RM14,357.24).
 2 RM94,357.24-RM80,000 = RM14,357.24.

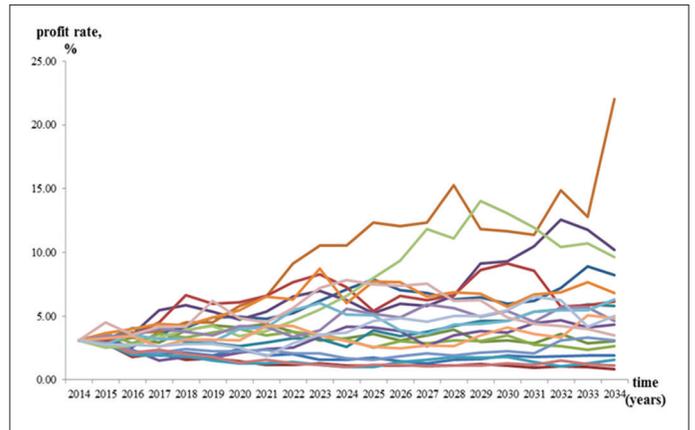
Table 4: Comparison of RMSE every model

Model	RMSE
Random walk Brownian motion with drift ¹	1.0503
Exponent random walk Brownian motion with drift ²	0.8126
Jump diffusion with drift ³	0.9935

$$^1 \frac{\delta S}{S} = -0.04(\delta t) + 0.2e\sqrt{\delta t}, \quad ^2 dS_t = -0.04S_t dt + 0.2S_t dW_t, \quad ^3 \frac{\delta S}{S} = -0.04(\delta t) + 0.2e\sqrt{\delta t} + 0.54F(0.1)\delta t$$

RMSE: Root mean square error

Figure 6: Pattern of 20 iteration



In addition, based on previous table, the maximum profit is RM58,327.73³. There is 99.8% chance to earn profits less than RM58,327.73. The bank probability of loss a lot of opportunities to earn profit less than RM58,327.73. It shows that the bank is offering too high profit. Therefore, the bank will burdern their borrower or customer to make payments.

About 90% chance for gaining profit more than RM10,884.34 and less than RM52,719.48. There is only 5% chance to earn profit less than RM10,884.34. It shows that the bank releases a lot of opportunities to earn profit. If the banks fix profits exceeding RM52,719.48, the banks will get too high profit. There is only 5% chance to earn profit more than RM52,719.48. Most of the time, the profit can fix lower than RM52,719.48. Therefore, probably customers will face difficulties to make payments because the bank fix too high profit.

This study suggests that the value in the middle of the distribution is the best and fair value to the bank and the customer. The median of the distribution is used as a profit benchmark which is RM30,565.06 (value at 50% certainty). So, the bank and the customer will facing equal risk. The bank will not set high profit that burden the customer. Furthermore, the bank also will not miss too much chance of lost profit.

To choose a profit benchmark, profit and probability were calculated for each iteration from Table 5. Table 7 shows selected iterations with profit and probability. These iterations can be considered as profit benchmark because the probability earning profit is near to the median. From this study, iteration 3 is suggested to be used as a benchmark. The profit to be gained by using the iteration 3 is RM29,201.35. The probability to get profit less than RM29,201.35 is 47%.

3 RM138,327.73-RM80,000 = RM58,327.73.

Table 5: Summary of forecast total installment with 20 iterations

Iteration	Total profit	Profit rate minimum	Profit rate maximum	Mean profit rate	Standard deviation profit rate
X_1	RM40,588.50	0.0257	0.0890	0.0575	0.0176
X_2	RM14,357.24	0.0083	0.0307	0.0143	0.0059
X_3	RM29,201.35	0.0284	0.0437	0.0341	0.0049
X_4	RM45,081.83	0.0293	0.1258	0.0681	0.0274
X_5	RM28,601.78	0.0258	0.0589	0.0389	0.0105
X_6	RM58,327.73	0.0307	0.2202	0.0961	0.0493
X_7	RM17,916.47	0.0129	0.0310	0.0196	0.0046
X_8	RM47,569.30	0.0307	0.0915	0.0626	0.0172
X_9	RM28,011.76	0.0236	0.0397	0.0317	0.0045
X_{10}	RM22,964.08	0.0152	0.0468	0.0319	0.0098
X_{11}	RM14,991.21	0.0101	0.0307	0.0161	0.0053
X_{12}	RM45,150.50	0.0307	0.0874	0.0603	0.0150
X_{13}	RM19,366.31	0.0160	0.0330	0.0233	0.0048
X_{14}	RM15,364.34	0.0103	0.0307	0.0156	0.0060
X_{15}	RM44,269.98	0.0252	0.1403	0.0732	0.0389
X_{16}	RM34,404.01	0.0307	0.0590	0.0452	0.0092
X_{17}	RM32,955.32	0.0283	0.0632	0.0439	0.0106
X_{18}	RM27,101.45	0.0247	0.0507	0.0340	0.0072
X_{19}	RM27,641.45	0.0186	0.0654	0.0394	0.0132
X_{20}	RM43,415.51	0.0307	0.0782	0.0531	0.0152

*Each iteration has 20 points of data (2014-2034) i.e., 20 years of forecasted values, **Yearly installment based on each year BFR rate

4.5. Argument of 4% by Using Conventional Method and BBA Method

If the interest rate is fix 4% by using conventional and BBA methods, the total profit can be obtained is RM37,730.80. Amortization tables calculated by using conventional method and BBA method are shown in Appendix Tables 2 and 3, respectively. The difference between the total profit by using 4% as an interest rate with the median profit stochastic forecasting is RM7,165.74. The difference between the profit by using 4% as an interest rate with profit by using iteration 3 is RM8,529.45 (RM37,730.80–RM29,201.35 = RM8,529.45).

5. CONCLUSION

One of the main concern in Islamic banks is to determine a profit benchmark of a house price. Profit is determined by using BFR. Therefore, the values of BFR are forecasted. BFR values cannot be fix with one value only because interest rates and return rates fluctuate, volatile and nonstationary. Stochastic forecasting is suitable to forecast a volatile and nonstationary value. The objective is to determine a profit benchmark by using stochastic forecasting.

There are several models in the stochastic forecasting which are a random walk Brownian motion with drift model, mean reversion with drift process, jump diffusion with drift process and mix process with drift are discussed. The finding is exponential random walk Brownian motion with drift is chosen for forecasting.

The forecasting results reveal the range of installment forecasting in 20 iterations is RM94,357.24 to RM138,327.73. In addition, the profit from the total minimum installment forecasting is RM14,357.24 and the profit obtained from the total maximum installment forecasting is RM58,327.73.

This study also found that the total installment distribution have a parabolic distribution with a minimum RM82,670.70

Table 6: The results of Kolmogorov test

Distribution	Test statistics	p value	Level	Estimated parameter value
Parabolic	0.11	0.97	1	$\alpha=82,670.75$ $\beta=140,128.60$
Lognormal	0.11	0.96	2	$\mu=112,370.50$ $\sigma=15,132.22$
Lognormal 3	0.11	0.96	3	$\gamma=112,370.60$ $\mu=15,132.30$ $\sigma=0.13$
Normal	0.12	0.94	4	$\mu=111,595.90$ $\sigma=14,419.61$
Cosine	0.12	0.92	5	$\alpha=84,859.23$ $\beta=138,327.70$

Table 7: Good iterations

Iteration	Total installment	Profit	Probability
X_3	RM109,201.35	RM29,201.35	$P(X<29,201.35)=0.47$
X_5	RM108,601.78	RM28,601.78	$P(X<28,601.78)=0.45$
X_9	RM108,011.76	RM28,011.76	$P(X<28,011.76)=0.44$
X_{16}	RM114,404.01	RM34,404.01	$P(X<34,404.01)=0.59$
X_{17}	RM112,955.32	RM32,955.32	$P(X<32,955.32)=0.56$
X_{18}	RM107,101.45	RM27,101.45	$P(X<27,101.45)=0.41$
X_{19}	RM107,641.45	RM27,641.45	$P(X<27,641.45)=0.42$

and maximum RM140,128.60. The profit rate was simulated 1,000 times and then the probability distribution of the profit rate was calculated. This study proposes RM30,565.06 as the best profit because this value in the middle of the distribution of profits. The best profit benchmark is determined by the median profit distribution.

The corresponding iteration has close to 50% chance of earning profit RM 30,565.06. In general, the findings of this study implies that the profit benchmark is fair to both bank and customer. In terms of statistical modelling, the use of stochastic methods in

forecasting and profit determination involves options to the bank for decision making.

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APPENDIX

Appendix Table 1: The amortization schedule for the first iteration based on the model proposed

Installment	Return of capital	Diminishing balance	Profit	Installment payment
A	B	C	D	E=B+D
0	0	80,000.00	40,588.50	-80,000.00
1	4000.00	76,000.00	2453.75	6453.75
2	4000.00	72,000.00	2413.02	6413.02
3	4000.00	68,000.00	1852.89	5852.89
4	4000.00	64,000.00	2529.11	6529.11
5	4000.00	60,000.00	2635.06	6635.06
6	4000.00	56,000.00	2945.54	6945.54
7	4000.00	52,000.00	2764.18	6764.18
8	4000.00	48,000.00	2488.76	6488.76
9	4000.00	44,000.00	2500.35	6500.35
10	4000.00	40,000.00	2715.03	6715.03
11	4000.00	36,000.00	2843.97	6843.97
12	4000.00	32,000.00	2841.52	6841.52
13	4000.00	28,000.00	2251.67	6251.67
14	4000.00	24,000.00	1912.10	5912.10
15	4000.00	20,000.00	1509.28	5509.28
16	4000.00	16,000.00	1300.34	5300.34
17	4000.00	12,000.00	955.27	4955.27
18	4000.00	8000.00	743.07	4743.07
19	4000.00	4000.00	577.45	4577.45
20	4000.00	0	356.14	4356.14
				120,588.50

Appendix Table 2: The amortization schedule with the interest rate is fixed at 4% on an annual basis using conventional methods

Installment	Return of capital	Diminishing balance	Profit (4%)	Installment payment
A	B	C	D	E=B+D
0	0.00	80,000.00	37,730.80	-80,000.00
1	4000.00	76,000.00	1886.54	5886.54
2	4000.00	72,000.00	1886.54	5886.54
3	4000.00	68,000.00	1886.54	5886.54
4	4000.00	64,000.00	1886.54	5886.54
5	4000.00	60,000.00	1886.54	5886.54
6	4000.00	56,000.00	1886.54	5886.54
7	4000.00	52,000.00	1886.54	5886.54
8	4000.00	48,000.00	1886.54	5886.54
9	4000.00	44,000.00	1886.54	5886.54
10	4000.00	40,000.00	1886.54	5886.54
11	4000.00	36,000.00	1886.54	5886.54
12	4000.00	32,000.00	1886.54	5886.54
13	4000.00	28,000.00	1886.54	5886.54
14	4000.00	24,000.00	1886.54	5886.54
15	4000.00	20,000.00	1886.54	5886.54
16	4000.00	16,000.00	1886.54	5886.54
17	4000.00	12,000.00	1886.54	5886.54
18	4000.00	8000.00	1886.54	5886.54
19	4000.00	4000.00	1886.54	5886.54
20	4000.00	0.00	1886.54	5886.54
				117,730.80

Source: Hasan (2011)

Appendix Table 3: The amortization schedule with the profit rate is fixed at 4% on an annual basis using the method of financing BBA

Installment	Return of capital	Diminishing balance	Profit (4%)	Installment payment
A	B	C	D	E=B+D
0	0.00	117,730.80		
1	2686.54	111,844.26	3200.00	5886.54
2	2794.00	105,957.72	3092.54	5886.54
3	2905.76	100,071.18	2980.78	5886.54
4	3021.99	94,184.64	2864.55	5886.54
5	3142.87	88,298.10	2743.67	5886.54
6	3268.59	82,411.56	2617.95	5886.54
7	3399.33	76,525.02	2487.21	5886.54
8	3535.30	70,638.48	2351.24	5886.54
9	3676.72	64,751.94	2209.82	5886.54
10	3823.78	58,865.40	2062.76	5886.54
11	3976.74	52,978.86	1909.80	5886.54
12	4135.80	47,092.32	1750.74	5886.54
13	4301.24	41,205.78	1585.30	5886.54
14	4473.29	35,319.24	1413.25	5886.54
15	4652.22	29,432.70	1234.32	5886.54
16	4838.31	23,546.16	1048.23	5886.54
17	5031.84	17,659.62	854.70	5886.54
18	5233.11	11,773.08	653.43	5886.54
19	5442.44	5886.54	444.10	5886.54
20	5660.13	0.00	226.41	5886.54
				117,730.80

Source: Hasan (2011). BBA: Bai 'Bithaman Ajil

Appendix Table 4: Installments when the gain is determined by the median

Installment	Return of capital	Diminishing balance	Profit	Installment payment
A	B	C	D	E=B+D
0	0	80,000.00	30,565.06	-80,000.00
1	4000.00	76,000.00	1528.25	5528.25
2	4000.00	72,000.00	1528.25	5528.25
3	4000.00	68,000.00	1528.25	5528.25
4	4000.00	64,000.00	1528.25	5528.25
5	4000.00	60,000.00	1528.25	5528.25
6	4000.00	56,000.00	1528.25	5528.25
7	4000.00	52,000.00	1528.25	5528.25
8	4000.00	48,000.00	1528.25	5528.25
9	4000.00	44,000.00	1528.25	5528.25
10	4000.00	40,000.00	1528.25	5528.25
11	4000.00	36,000.00	1528.25	5528.25
12	4000.00	32,000.00	1528.25	5528.25
13	4000.00	28,000.00	1528.25	5528.25
14	4000.00	24,000.00	1528.25	5528.25
15	4000.00	20,000.00	1528.25	5528.25
16	4000.00	16,000.00	1528.25	5528.25
17	4000.00	12,000.00	1528.25	5528.25
18	4000.00	8000.00	1528.25	5528.25
19	4000.00	4000.00	1528.25	5528.25
20	4000.00	0	1528.25	5528.25
				110,565.06

Appendix Table 5: Comparison of equation among all methods

Method	Installment	Diminishing balance equation	Total paid
Conventional	$R = \frac{i(1+i)^n A}{(1+i)^n - 1}$	$DB_t = R \left(\frac{1-v^{n-t}}{i} \right)$	$\sum_{t=1}^n t \left(\frac{i(1+i)^n A}{(1+i)^n - 1} \right)$
BBA	$R = \frac{i(1+i)^n A}{(1+i)^n - 1}$	$DB_t = Rn - \sum_{i=0}^t R_i$	$\sum_{t=1}^n t \left(\frac{i(1+i)^n A}{(1+i)^n - 1} \right)$
MMP	$R = \frac{x(1+x)^n A}{(1+x)^n - 1}$	$DB_t = DB_{t-1} - \left[\left(\frac{x(1+x)^n A}{(1+x)^n - 1} - \text{rental} \right) + (p \times \text{rental}) \right]$	$\sum_{t=1}^n t \left(\frac{x(1+x)^n A}{(1+x)^n - 1} \right)$
ZDBM	$R_t = \frac{A}{n} + (DB_{t-1} \times \text{profit rate})$	$DB_t = DB_{t-1} - \frac{A}{n}$	$\sum_{t=1}^n t \left(\frac{A}{n} + (DB_{t-1} \times \text{profit rate}) \right)$
Suggestion	$R_t = \frac{A}{n} + \left(\frac{\text{Total profit}}{n} \right)$	$DB_t = DB_{t-1} - \frac{A}{n}$	$\sum_{t=1}^n t \left(\frac{A}{n} + \left(\frac{\text{Total profit}}{n} \right) \right)$

Where, R : Periodic payment, i : Periodic interest rate, n : Tenure, A : Present value of annuity at $t=0$ /Total loan, DB_t : Diminishing balance at time t , a_{n-1} : Present value between tenure ($t-1$), t : Payment time at t , $v = \frac{1}{1+i}$, DB_{t-1} : Diminishing balance at time $t-1$, x : Rental and asset price ratio $\left(\frac{S}{P} \right)$, p : Customer's ownership ratio $\left(\frac{\text{customer's equity}}{A} \right)$. Source: Shafiea and Isa (2015). BBA: Bai 'Bithaman Ajil, MMP: Musharakah Mutanaqisah Partnership, ZDBM: Zubair diminishing balance model