

Random or Deterministic? Evidence from Indian Stock Market

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ABSTRACT

This study investigates the presence of long memory and non-linear dynamics in Indian stock market returns for a period of 19 years from May 1997 to May 2016 by using rescaled range $\binom{R}{S}$ method and V-statistics. The empirical findings suggest that Indian stock market shows a high degree of long-range persistence and future stock price can be predicted. The study also finds the presence of multiple non-periodic cycles in the data generating process, with a maximum cycle length of 3.7 years. This study is quite helpful to the participants of the capital markets to improve their portfolio performance by taking efficient strategy before making investment decision.

Keywords: $\frac{R}{S}$ Analysis, V-statistic, Non-linear Dynamics **JEL Classifications:** C22, C53, G14

1. INTRODUCTION

The presence of long memory in stock returns is one of the popular research topic in finance today. The presence of long memory in stock return indicates that the market does not respond immediately to information flows into the market but reacts gradually over time. Long range dependence reflects the existence of predictable parameter in a dynamic time series because it associates the assets future returns to previous returns. But however the presence of long memory in stock return contradicts the weak form of market efficiency hypothesis. Fama, 1970 proposed efficient market hypothesis (EMH) and the main content is that in the efficient market, the price includes only the information set of past prices. In the semi-strong efficient market, the price includes the previous price information and other shared information. In the strong efficient market, prices include the exclusive information of market subset, and all shared information. Hence no one could outperform the market by using the same information discarding to all investors. Ascertaining the patterns of price changes cannot be used to predict the future value of assets since news are unpredictable, price changes are unexpected and the returns are realizations of a random process. In last couple of years, some studies have found lot of opposite surprise that EMH cannot make a reasonable explanation. It indicates that EMH based on the classical capital market theory has many imperfections and need to be continuously developed. Keeping in view, the estimation of the long memory parameter is particularly important at present concerns. Long memory reflects the existence of predictable parameter in a dynamic time series because it associates the assets future returns to previous returns. It has been argued that the presence or absence of long memory in the stock returns provides the validity of EMH and guidelines of market efficiency. Further, the identification of market cycles give rise to an opportunity to earn abnormal returns in the stock markets. Therefore, exploring long memory property becomes a topic of interest among traders, policy makers, academicians and researchers today. In the last three decades, emerging markets became a viable alternative for investors seeking international diversification. Among the emerging markets, India has received productive share of foreign investment inflows since last couple of years. However, there is little research work has been done to determine the presence of long memory in Indian stock market, This motivates us for exploring research in Indian market to position country's exposure to the outer world. Keeping in view the present research study has taken to fill this gap. The study has raised two research questions. First, the present study will add to the existing literature by providing robust result. Secondly the study has used rescaled range analysis to examine the evidence of long memory and some type of non-periodic cycles in the Indian stock market returns. The paper is organized as follows. Section II reviews the existing literature, Section III discusses the data and methodology, Section IV describes the empirical findings and Section V presents conclusions.

2. LITERATURE REVIEW

The application of Hurst exponent in finance to investigate non-linear structure is carried out by some studies. Peters (1989) applies $\frac{R}{S}$ analysis to unveil masked fractal nature of asset returns, using stock return data from 1950 to 1988. The findings suggest that stock returns follow a biased random walk: They tend to move in one direction until any exogenous factors change their bias. Moreover, returns don't strictly follow EMH, suggesting lagged reflection of investors' interpretation on asset prices. Even in the presence of considerable white noise in return series, he concludes, it is possible to forecast them. Scheinkman and LeBaron (1989) find evidence of non-linear dynamics in weekly returns of CRSP value-weighted index data and find that returns series do not follow a random walk, as suggested by EMH. Lo (1991) modifies the classical $\frac{R}{S}$ method, which is robust for detecting short run dependence, to account for long memory. However, unlike the previous studies, he finds little evidence of long memory in US market returns. Peters (1992; 1994) use $\frac{R}{S}$ Analysis to S and P 500 index monthly returns and Dow Jones Industrial returns, respectively, and the evidence suggests the presence of long-range dependence and cycles in both the returns series. Cheung and Lai (1995) finds no evidence of long-term dependence in stock index returns of 18 countries. Similarly, Jacobsen (1996) finds no evidence of long memory in the index return data of five European countries, applying Lo's modified $\frac{R}{S}$ method. Opong et al. (1999) find substantial evidence against random behavior in return series of four FTSE indices. Additionally, the return series are found to exhibit some cycles, which appear more frequently than would be expected in a truly random series. Huang and Yang (1999) use modified $\frac{R}{S}$ technique to determine the long memory in NYSE and NASDAQ indexes. The study confirms the presence of long memory in the market. Howe et al. (1999) use both $\frac{R}{S}$ and its modified versions, to market data of Japan, Australia, Hong Kong, Taiwan, Singapore and Korea and their results show that although classical $\frac{R}{S}$ method indicates presence of nonrandomness, after applying the modified version, the long-run dependence structure disappears. This study asserts that there is no presence of long memory in the asset market, which is consistent with EMH. Chen (2003) investigates the long memory in Shanghai Stock Index and Shenzhen Stock Index by using modified $\frac{R}{S}$ method. The study finds these market are not significantly long term related. Cajueiro and Tabak (2004) use Hurst exponent and find the evidence of long-range dependence for Australia, Hong Kong, Singapore, and Japan in both equity returns and volatility. Assaf and Cavalcante (2005) use both Lo' $\binom{R}{S}$ statistics and FIGARCH model to find long memory in return and volatility of Brazilian Stock Market. The study finds that there is no long memory presence in Brazilian stock market.

Assaf (2006) establishes similar results for asset market of Kuwait, which shows the existence of persistence and hence rejects the EMH. Studies on Indian market forward mixed results too. Selvam et al. (2011) using BSE Sensex data and shows that the index initially follows a random walk, which later becomes persistent. Mahalingam et al. (2012) finds strong evidence of long memory in monthly return series of BSE Sensex, and CNX 500 daily return series, respectively. Gunay (2015) tests the existence of long memory in index returns of BRIC countries and finds positive results.

The existing literature provides conflicting evidence on a long memory of asset returns. Hence it motivate us to undertake the study to determine the non-linear departure from the random-walk behavior of stock returns by using $\frac{R}{S}$ method. The present study contributes to the investigation of long memory in emerging markets, as these markets are said to be the most likely places for assets to exhibit long memory, due to their not-fully-developed capital market structure.

Liu, (2000) examines long memory of daily S&P returns by using regime switching stochastic volatility (RSSV) model. The study finds that past returns can be generated by projection. The study also finds the presence of short memory component of volatility over and above the long memory.

3. DATA AND METHODOLOGY

The required time series daily closing prices of BSE Sensex have been collected from July 1997 to June 2016 from the official webpage of Bombay Stock Exchange (www.bseindia.com). Returns are proxies by the log difference change in the price index.

For the data set daily returns are calculated for BSE Sensex by using the following formula.

$$Rt = ln (Pt/Pt-1)*100$$
 (1)

Pt is the current price and Pt-1 is the previous day price

The study used Jarque-Bera test to determine whether Indian stock market follows the normal probability distributions.

3.1. R/ Analysis Method

Mandelbrot and Wallis (1969) and Hurst (1951) introduced rescaled range $\binom{R'_S}{S}$ statistics to decide whether long memory exist or not. $\frac{R'_S}{S}$ statistics (H) measures the intensity of long-range dependence in a time series. Rescaled range analysis is a statistical procedure used to identify and assess the persistence, randomness, or mean reversion in time series data.

Suppose $\{X_j\}_1^N$ represents a time series of asset returns recorded over discrete time period N, with mean return \overline{X} . The series is divided into k non-overlapping sub-samples of length d, where d

is the integer part of N/k¹ Now fixing $t_i=d(i-1)+1$ as the starting points of the sub-samples, where the condition $(t_i-1)+d \le N$ is satisfied, we construct a new time series W(i,k), which is nothing but accumulated deviations from sample means for all the k subsamples.

$$W(t_{i},k) = \sum_{j=1}^{k} [X_{(t_{i}+j-1)} - \frac{1}{d} \sum_{j=1}^{d} X_{(t_{i}+j-1)}], k=1...d$$
(2)

The widest difference range R is calculated as the difference between maximum and minimum accumulated deviation from the mean, as follows:

$$R(t_{i},d) = \max[0,W(t_{i},1),W(t_{i},2)....W(t_{i},d)] - \min[0,W(t_{i},1), W(t_{i},2)....W(t_{i},d)]$$
(3)

When the above range is rescaled with the standard deviation of the series $S(t_i,d)$ we get $\frac{R(t_i,d)}{S(t_i,d)}$ statistic for a number of values of d. The standard deviations for the samples $X_{t_i}, X_{t_i}, \dots, X_{t_i+d-1}$ are calculated as:

$$S(t_{i},d) = \sqrt{\frac{1}{d} \sum_{j=1}^{d} [X_{(t_{i}+j-1)} - \frac{1}{d} \sum_{j=1}^{d} X_{(t_{i}+j-1)}]^{2}}$$
(4)

Thus, for each value of d, we get some $\frac{R}{S}$ samples, and the number of samples declines as the value of d increases for the limiting condition put on the values that d can take. Hurst noticed that the observed rescaled range $\frac{R}{S}$ for many natural phenomena, with sample size n is fittingly represented by the empirical equation:

with sample size n is fittingly represented by the empirical equation:

$$E(\frac{R}{S}) = Cn^{H}, as n \to \infty....$$
(5)

Where C is proportionality constant and H is the Hurst exponent, and the value it takes is believed to govern the persistent structure of a time series.

Modifying equation 1 as follows, the Hurst component H is calculated as the coefficient of the term \log_{10} in equation 2.

$$\log_{10}(\frac{R}{S}) = \log_{10}C + H \log_{10}n$$
 (6)

The plot of $\log_{10}(\frac{R}{S})$ against \log_{10} n gives us what is called pox diagram. Fitting a least square line through the pox plot, we get the Hurst exponent H as the slope of the fitted line. The smallest values of d in the $\frac{R}{S}$ samples are dominated by short-run correlations and hence are not considered, and the samples with an enormous value of d are considered statistically insignificant if some samples per d are less than say 5 (Rose, 1996). The computed value of H indicates the presence of persistence or long memory patterns in the considered data series.

A precise definition of long memory is crucial before using Hurst component to identify such a process. A stationary discrete-time processes X is said to possess long-range dependence, or long memory, when its auto-covariance function $\gamma(\cdot)$ decays so slowly that $\sum_{k=0}^{\infty} (\gamma_k)$. Contrast to this is short memory processes, where

process covariances are summable. Whether a process is random, persistent, or not can be determined by looking at the values of H. In general, the following statements are empirically proven and theoretically sound

- If H=0.5, the data series represents a random process
- If H>0.5, the series exhibits persistent dependence, in terms of large values following large ones and vice-versa
- If H<0.5, a process with mean reversion is observed

3.2. V Statistic

The V-statistic is used to confirm the presence of long-range dependence in the data series. Additionally, it also helps discerning any cyclic behavior in the data. Following McKenzie (2001), the statistic and its properties are described as follows:

V Statistic_n =
$$\frac{\left(\frac{R}{S}\right)_n}{\sqrt{n}}$$
, for sample of size n. (7)

V-statistic is calculated for both $\binom{R'_S}{S}$ and $E\binom{R'_S}{S}$, and the values are compared to investigate the presence of long memory in data series. This comparison is based on the fact that, in the $\binom{V-\text{statistic}}{\log(n)}$ space; the plot of V-statistic against (log[n]) will roughly be a straight line, if $\binom{R'_S}{S}$ are IIDs. As $E\binom{R'_S}{S}$ are IIDs, the plot of V-statistic of $E\binom{R'_S}{S}$ will largely be a smooth line. In order to observe behavior of $\binom{R'_S}{S}$ that is different from that of an IID, we compare the V-statistics of $\binom{R'_S}{S}$ with that of $E\binom{R'_S}{S}$. If the data generating process is persistent (anti persistent), then $\binom{R'_S}{S}$ will change at faster (slower) pace than (\sqrt{n}), and will have positive (negative) slope (Peters. 1994).

4. EMPIRICAL ANALYSIS

Before applying $\frac{R}{S}$ analysis method, it is important to evaluate the structure of the data, and remove any linear dependence structure, if presents in the data series. We fit an autoregressive

Table 1: Descriptive statistics of daily returns of BSE sensex

Fields	Original return series	Residuals of AR model
Counts	4682	4682
Mean	0.000390479	-1.55775E-07
Maximum	0.159899849	0.157744073
Minimum	-0.118091758	-0.113935296
SD	0.015765246	0.015724606
Variance	0.000248543	0.000247263
Skewness	-0.105687713	-0.047311797
Kurtosis	6.028457532	5.931254529
Range	0.277991607	0.271679369

SD: Standard deviation

N & k are chosen in a way that N/k is always an integer.

(AR) model to the data set and apply $\frac{R}{S}$ method to the residual series of the AR model. The AR model doesnot ensure complete elimination of linear dependence, reduces such dependence to a very insignificant level (Brock et al., 1996). The descriptive statistics in Table 1, for both the original and whitened returns series, reports Skewness and excess kurtosis of (-0.105687713, 6.028457532) and (-0.047311797, 5.931254529) respectively. It is observed that the whitened series is less skewed and leptokurtic as compared to the original series.

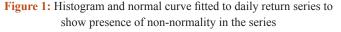
To test the normality of the whitened series, we perform Jarque Bera test. The JB test statistic value is 6847.1, with P < 2.2e-16. It is observed that the test statistic exceeds the critical values, and the null hypothesis is rejected. It indicates that the BSE stock returns follow a non-normal distribution. So it indicates that greater the fat-tail degree means the stronger persistence, and more important of the historical information that may forecast price trends.

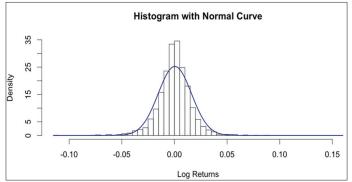
Figure 1 presents the histogram of the return series, and a normal curve fitted to the histogram. The presence of non-normality in data indicates some systematic bias in the data generating process, which can be identified by using $\frac{R}{S}$ analysis method.

Before applying the $\frac{R}{S}$ method, we initially divide the data into n non-overlapping sub-periods, where n is an integer, which evenly divides the data series. Since we have total 4682 data points, we reduce data points to generate maximum numbers of factors, and maximum number $\frac{R}{S}$ values. We consider 38 factors for our study, with a starting value of 12. The elements are chosen in such a way that n > 10 always holds true. This is done to ensure that the results obtained are accurate, as values smaller than ten results in unstable estimates (Peters, 1994).

Each non-overlapping sub-periods are calculated by following standard procedures range and then these ranges are normalized by dividing with individual standard deviations. The average $\frac{R}{S}$ values and expected values of each $\frac{R}{S}$, for 38 sub-periods, are calculated and presented in Table 2. To support the interpretation of these estimates, Figure 2 presents a plot of $\log(\frac{R}{S})$ and $\log E\left(\frac{R}{S}\right)$ against $\log(n)$.

It is observed from Figure 2 that $\log (\frac{R}{S})$ and $\log E (\frac{R}{S})$ move parallel to each other till the point 1.8921 (n = 78), and show systematic deviations from each other beyond this point. In the interval between the points 1.8921 (n = 78) and 2.9713 (n = 936), $\frac{R}{S}$ doesn't follow random walk like the $E (\frac{R}{S})$. Moreover, the presence of breakpoints, which indicates evidence of cycles, can be seen at points 2.1939, 2.4150, 2.9713. The presence of breaks that are not evident from Figure 2, is detected with the help of a plot of V-statistic for $\frac{R}{S}$ and $E (\frac{R}{S})$ against Log (n). The plot of the V-statistic is exhibited in Figure 3. V-statistic displays whether the return series is persistent or not. If the V-statistic of $\frac{R}{S}$ increases at a faster pace than that of the $E (\frac{R}{S})$, we conclude





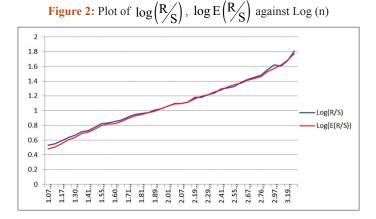
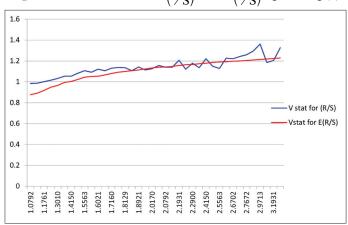


Figure 3: Plot of V-statistic for $\binom{R}{S}$ and $E\binom{R}{S}$ against Log (n)



that the underlying process of data series is persistent. From the Figure 3, we can see that $\frac{R}{S}$ is increasing at faster rate than $E\left(\frac{R}{S}\right)$; we can thus conclude that the data generating process of BSE Sensex daily returns is persistent.

The cyclic behavior of the process is analyses and revealed in Figure 3. Figure 3 shows the existence of multiple cycles in the data series, thereby complementing the evidence obtained from Figure 2. The existence of period is indicated by changes in the directions of V-statistic for $\left(\frac{R}{S}\right)$; a downward move from the

Table 2: Results of the rescaled	l range analysis for daily	Indian stock market return

Sub-period	R/S	R	$Log_{10}(n)$	V-statistic	$\mathbf{E}(\mathbf{R}/\mathbf{)}$	R.	E (V-statistic)
Lengths (n)	/ S	$\log_{10}(\frac{R}{S})$			$\mathbf{E}\left(\mathbf{R}_{S}\right)$	$\log_{10}(\mathrm{E}(\frac{\mathrm{R}}{\mathrm{S}}))$	
12	3.41353	0.53320	1.07918	0.98540	3.03739	0.48250	0.87682
13	3.56274	0.55178	1.11394	0.98813	3.21853	0.50766	0.89266
15	3.87918	0.58874	1.17609	1.00160	3.56053	0.55151	0.91933
18	4.31458	0.63494	1.25527	1.01696	4.03233	0.60556	0.95043
20	4.61609	0.66427	1.30103	1.03219	4.32474	0.63596	0.96704
24	5.16367	0.71296	1.38021	1.05403	4.86773	0.68733	0.99362
26	5.37483	0.73036	1.41497	1.05409	5.12185	0.70943	1.00448
30	5.93526	0.77344	1.47712	1.08363	5.60177	0.74833	1.02274
36	6.63721	0.82199	1.55630	1.10620	6.26415	0.79686	1.04403
39	6.82579	0.83415	1.59106	1.09300	6.57424	0.81785	1.05272
40	7.09928	0.85121	1.60206	1.12249	6.67489	0.82444	1.05539
45	7.43345	0.87119	1.65321	1.10811	7.16000	0.85491	1.06735
52	8.15123	0.91122	1.71600	1.13037	7.79530	0.89183	1.08101
60	8.81948	0.94544	1.77815	1.13859	8.47037	0.92790	1.09352
65	9.15496	0.96166	1.81291	1.13553	8.86932	0.94789	1.10010
72	9.38568	0.97247	1.85733	1.10611	9.40262	0.97325	1.10811
78	10.08902	1.00385	1.89209	1.14236	9.83923	0.99296	1.11407
90	10.56835	1.02401	1.95424	1.11400	10.66429	1.02793	1.12411
104	11.49442	1.06049	2.01703	1.12712	11.55951	1.06294	1.13350
117	12.52632	1.09782	2.06819	1.15806	12.33781	1.09124	1.14063
120	12.49769	1.09683	2.07918	1.14088	12.51113	1.09730	1.14211
130	13.00450	1.11409	2.11394	1.14057	13.07367	1.11640	1.14664
156	15.07450	1.17824	2.19312	1.20693	14.44173	1.15962	1.15626
180	15.03953	1.17723	2.25527	1.12098	15.60585	1.19329	1.16319
195	16.44529	1.21604	2.29003	1.17767	16.29407	1.21203	1.16684
234	17.36289	1.23962	2.36922	1.13505	17.96800	1.25450	1.17460
260	19.70580	1.29459	2.41497	1.22210	19.00695	1.27891	1.17876
312	20.29936	1.30748	2.49415	1.14923	20.93887	1.32095	1.18543
360	21.40724	1.33056	2.55630	1.12826	22.58313	1.35378	1.19024
390	24.19899	1.38380	2.59106	1.22536	21.69949	1.33645	1.09880
468	26.40998	1.42177	2.67025	1.22080	25.92024	1.41364	1.19816
520	28.36206	1.45274	2.71600	1.24376	27.38825	1.43756	1.20105
585	30.47061	1.48388	2.76716	1.25980	29.12338	1.46424	1.20410
780	36.16170	1.55825	2.89209	1.29480	33.81617	1.52912	1.21081
936	41.66678	1.61979	2.97128	1.36192	37.15890	1.57006	1.21458
1170	40.56326	1.60813	3.06819	1.18588	41.68685	1.62000	1.21873
1560	47.59484	1.67756	3.19312	1.20503	48.32120	1.68414	1.22342
2340	64.14097	1.80714	3.36922	1.32595	59.44931	1.77415	1.22896

previous upward move is marked as starting of the cycle. The analysis indicates that underlying data generating process is deterministic embedded with periodic cycles.

In the absence of any cyclic patterns in the data making process, the plot of V-statistic against Log (n) gives a positive slope. It is observed that V-statistic shows positive slope since there are non-periodic cycles present in the stock returns. From Figure 3, it is evident that till the point 1.8921 (n = 78), the data series follows a random walk process. After n = 78 the V-statistic of $\frac{R}{S}$ deviates from that of $E\left(\frac{R}{S}\right)$, with systematic breaks evident at five points. For n > 936 values of both the V-statistics tend to converge. Table 3 presents the break points with the number of days/years at which the breaks happen.

Table 3 presents the five cycles identified in years is 0.46, 0.61, 1.03, 1.55 and 3.71 years.

The discerning cyclic patterns with help of Hurst component (H) is attained by calculating it in the range (n = 78, n = 936). The

Table 3: The break points

$Log_{10}(n)$	Days (n)	Year (considering 252 trading days)
2.0682	117	0.464
2.1931	156	0.619
2.4150	260	1.032
2.5911	390	1.548
2.9713	936	3.714

Hurst component value for each cycle is presented in Table 4.

Table 4 shows the Hurst component value (H) for different intervals where breakpoints or cycles are identified. For the first interval, $78 \le n \le 936$, value of H for $\frac{R}{S}$ is 0.56068, and that for $E\left(\frac{R}{S}\right)$ is 0.534321. The value of H for $\frac{R}{S}$ in this range is significant at 0.001, implying that Indian stock market return series follows random walk is rejected. It exhibits the evidence of persistence in the return generating process. Out of the overall intervals, the last entry 468 and 963 provides the evidence of strong persistence in the return series, at a significance level of 0.001. At two intervals (78, 117) and (180, 260) the series exhibits

Interval	H (R/S)	t-statistic	Р	Adjusted R ²	$H E \left(\frac{R}{S} \right)$	t-statistic	Р	Adjusted R ²
(78,936)	0.56068 (0.01037)	54.1	<2e-16***	0.994	0.53432 (0.00154)	346.7	< 2e-16***	0.9999
(78,117)	0.53538 (0.06259)	8.6	0.0134*	0.960	0.55812 (0.00143)	388.3	6.63e-06***	1
(120,156)	0.73095 (0.07305)	10.0	0.0634**	0.980	0.54676 (0.00084)	649.0	0.000981***	1
(180, 260)	0.6502 (0.1255)	5.2	0.0353*	0.896	0.53616 (0.00063)	846.7	1.39e-06***	1
(312, 390)	0.7385 (0.3002)	2.5	0.246	0.716	0.52773 (0.000437)	1206.2	0.000528***	1
(468, 936)	0.64417 0.01789	36	4.71e-05***	0.997	0.51957 (0.000529)	982.45	2.33e-09***	1

Significance level: ***: 1%, **: 5%, *:10%

deviations from a random walk, with values of H as 0.535 and 0.65 respectively (significance level of 0.05). Lastly at 0.1 level of statistical significance, in the interval (120, 156), we observe a high value of H (0.73095). For $312 \le n \le 390$, the Hurst component assumes a value of 0.7385; however this value is not statistically significant revealing the non -persistent pattern in the return series. The presence of multiple non-periodic cycles ranging from about 0.5 to 3.7 years is evident in the return series.

If the values of H for $\frac{R}{S}$ and $E\left(\frac{R}{S}\right)$ is compared, it clearly shows that the former one increases at a faster pace as compared to the random process of the latter one. The analysis suggests that the Indian stock market return series has the presence of long-range dependence.

5. CONCLUSIONS

This study has investigated the presence of long memory in Indian stock market by taking the non-parametrical statistics including $\frac{R}{S}$ method and $\frac{V}{S}$ method. The study finds that the skew ness of daily return is positively skewed and kurtosis is greater than 3 indicating fat tail trait. So it indicates that Indian stock market has not normally distributed and it is not like that in the traditional effective markets in which time series is random.

The present study finds that Indian stock market return series exhibits persistent behavior. The study also finds the multiple non-periodic cycles with a maximum value of 3.7 years, which indicates the presence of non-linear dynamics in the market. The implications of such evidence are of high relevance to policy makers and all capital market participants.

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